XX SERBIAN ASTRONOMICAL CONFERENCE
16-20 October 2023, SASA, Belgrade

FAST NUMERICAL TEST OF INFLATIONARY MODELS

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This work has been supported by the Serbian Ministry for Education, Science and Technological Development under contract **No. 451-03-47/2023-01/ 200124.**

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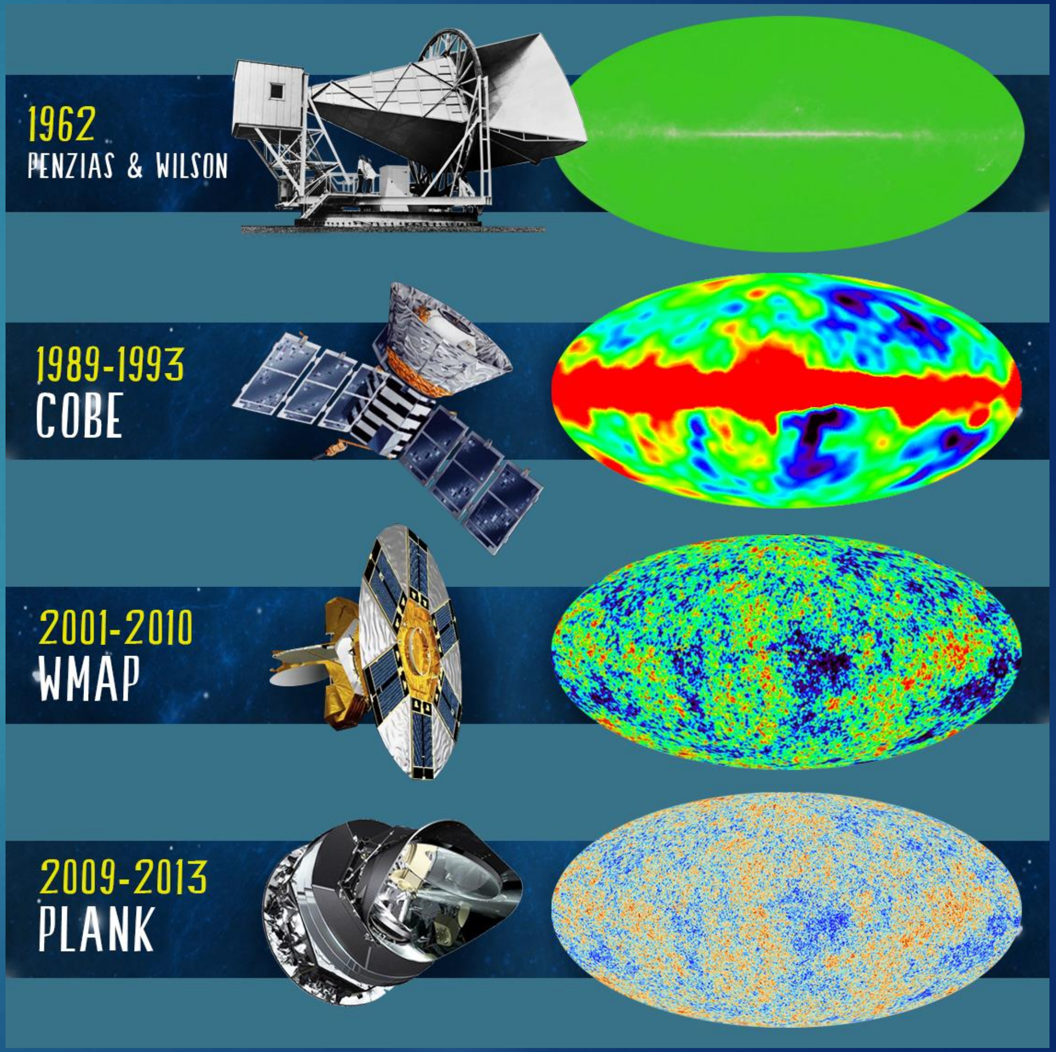
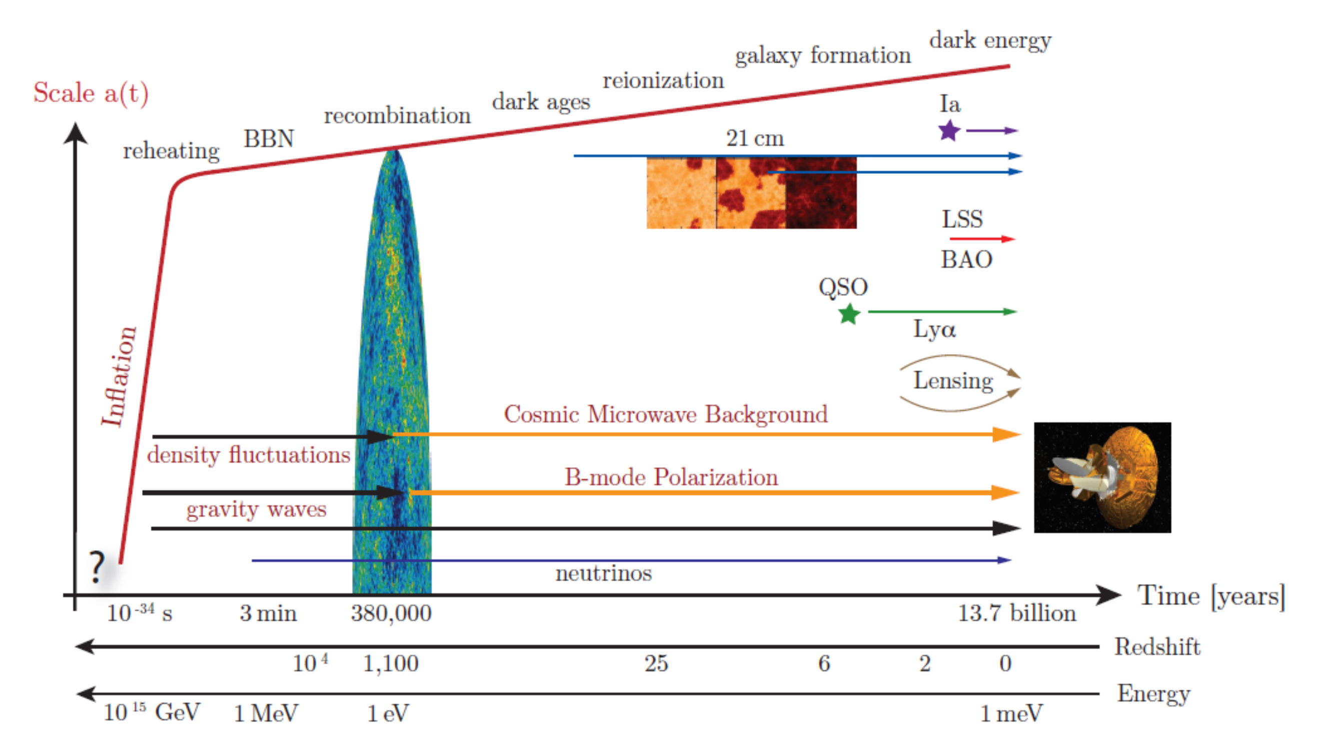


Introduction

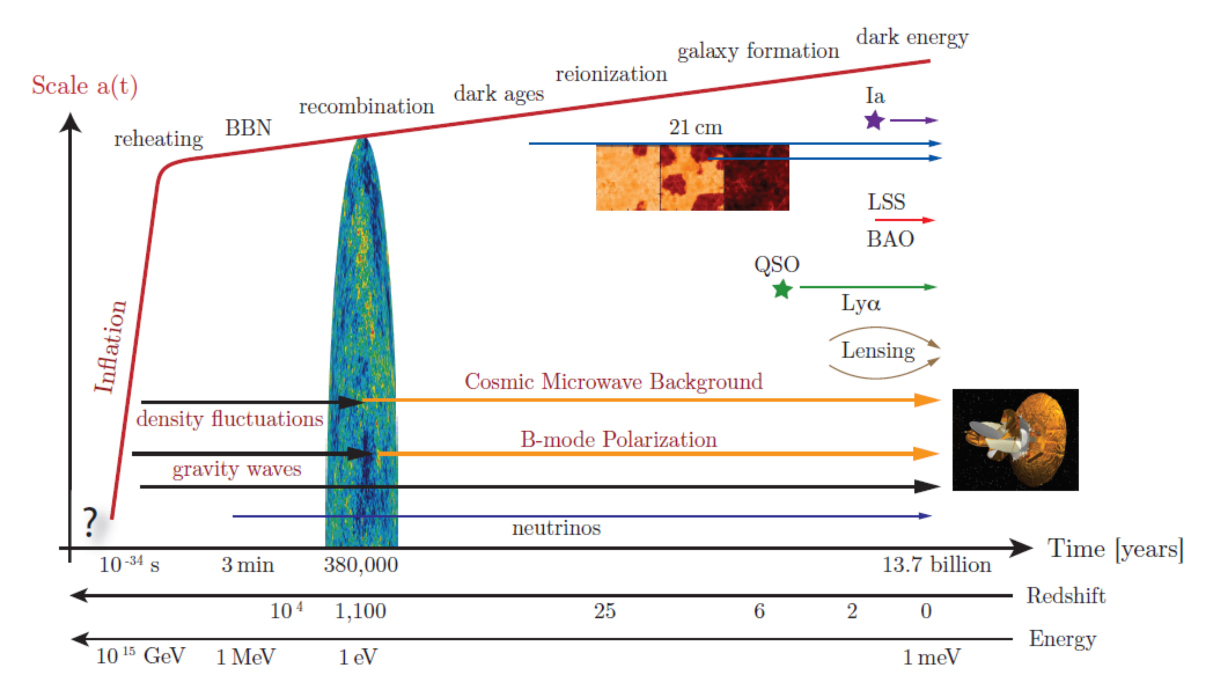
- The ***inflation theory*** proposes a period of extremely rapid (exponential) expansion of the universe during the an early stage of evolution of the universe.
- The inflation theory predicts that during inflation (it takes about $10^{-34}\ s$) radius of the universe increased, at least $e^{60} \approx 10^{26}$ times.
- Although inflationary cosmology has successfully complemented the Standard Model, the process of inflation, in particular its origin, is still largely unknown.



Baumann, D. TASI Lectures on Inflation. (2009), arXiv:0907.5424 [hep-th]

Introduction

- Over the past 40 years numerous models of inflationary expansion of the universe have been proposed.
- The simplest model of inflation is based on the existence of a single scalar field, which is called ***inflaton***, which drives inflation.
- Recent years brought us a ***lot of evidence*** from WMAP and Planck observations of the CMB
- The most important way to ***test inflationary cosmological models*** is to compare the computed and measured values of the ***observational parameters.***
- We present a computational method for **fast numerical testing of different models** based on standard single field and tachyon inflation



Baumann, D. TASI Lectures on Inflation. (2009), arXiv:0907.5424 [hep-th]

The Homogenous and Isotropic Universe

- The Friedmann-Robertson-Walker (FRW) metric:

$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right],$$

where $\mathrm{a}(t)$ is the scale factor, k is the spatial curvature parameter (+1,0,-1).

- Comoving coordinates – the universe expands as $\mathrm{a}(t)$ increases, however galaxies stay at fixed coordinates r, θ, ϕ.
- The physical (real) distance is time-dependent even for object with zero peculiar velocities, i.e.

$$\vec{X} = \mathrm{a}(t)\,\vec{r}$$

Dynamics of the Universe

$$T^{\mu}_{\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

- It is determinated by the Einstein equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu},$$

where $R_{\mu\nu}$ and R are Ricci tensor and scalar, and T^{μ}_{ν} is energy-momentum tensor .

- The Einstein Equations --> two coupled the **Friedman equations**

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

where $p = \mathcal{L}$ is pressure, and $\rho = \mathcal{H}$ is energy density.

Standard Single Field Inflation

- The simplest models - standard single scalar field inflation, a field ϕ - ***inflaton***
- A condition for inflation (from the Friedmann equations)

$$\frac{d}{dt}(aH)^{-1} < 0 \quad \Leftrightarrow \quad \frac{d^2a}{dt^2} > 0 \quad \Leftrightarrow \quad \rho + 3p < 0$$

A negative pressure runs inflation

- The dynamics of the classical real scalar field

$$S = -\frac{1}{16\pi G}\int \sqrt{-g}\, R d^4x + \int \sqrt{-g}\, \mathcal{L}(X,\varphi) d^4x,$$

- where $\mathcal{L}(X,\varphi)$ is the Lagrangian, with kinetic term $X \equiv \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi$
- Energy density and pressure

$$\rho \equiv \mathcal{H} = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p \equiv \mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

the canonical Lagrangian

Standard Single Field Inflation

- Time evolution of homogeneous scalar field, for FRW metric → the Klein–Gordon equation

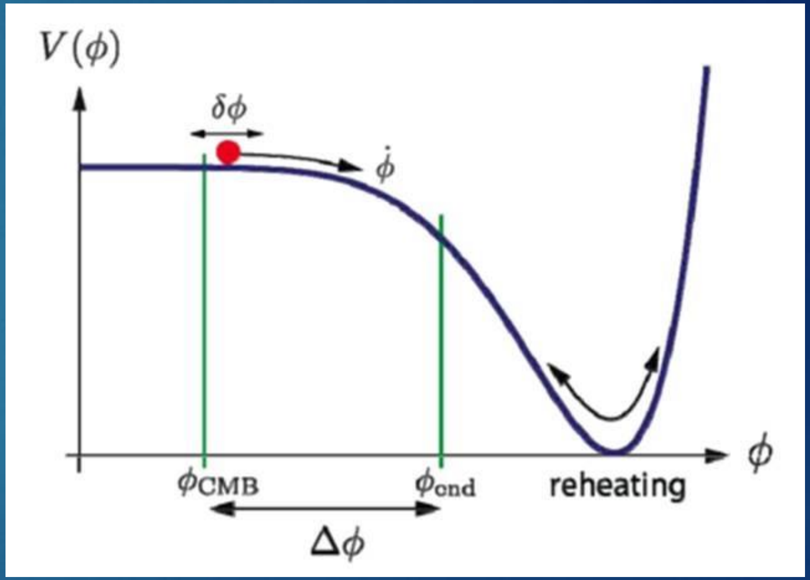
$$\ddot{\phi} + 3H\dot{\phi} + V' = 0, \qquad V' \equiv \frac{\partial V}{\partial \phi}$$

- The Friedmann equation

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

- Slow-roll condition

$$\dot{\phi}^2 \ll V(\phi) \Rightarrow \begin{cases} H^2 \approx \dfrac{8\pi G}{3} V(\phi) \\ 3H\dot{\phi} + V' \approx 0 \end{cases}$$



- In order for inflation to last long enough

$$|\ddot{\phi}| \ll |3H\dot{\phi}|$$

$$|\ddot{\phi}| \ll |V'|$$

Hamilton's equations

- Definition:

$$\frac{d\phi}{dt} = \frac{\partial \mathcal{H}}{\partial \pi}, \quad \frac{d\pi}{dt} = -3H\pi - \frac{\partial \mathcal{H}}{\partial \phi}$$

where $\mathcal{H} = \mathcal{H}(\phi, \pi, t)$, $\mathcal{L} = \mathcal{L}(\phi, \dot{\phi}, \mathrm{t})$ and $\pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$ is a conjugate momenta

- Recall

$$\rho \equiv \mathcal{H} = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p \equiv \mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\dot{\phi} = \pi$$
$$\dot{\pi} = -3H\pi - V'$$

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V\right)$$
$$\dot{H} = -\frac{8\pi G}{2}\dot{\phi}^2$$

$$\theta_{,\mu} = \frac{\partial \mathcal{H}}{\partial \pi^\mu}$$
$$\pi^\mu_{;\mu} = -\frac{\partial \mathcal{H}}{\partial \theta}$$

Observational parameters

$H \equiv \frac{\dot{a}}{a}$

- ***Hubble hierarchy (slow-roll) parameters***

$$\epsilon_{i+1} \equiv \frac{d\ln|\epsilon_i|}{dN} = \frac{d\ln|\epsilon_i|}{d\ln a}, \quad i \geq 0, \quad \epsilon_0 \equiv \frac{H_*}{H}$$

Hubble expansion rate at an arbitrarily chosen time

$$\varepsilon_1 = -\frac{\dot{H}}{H^2},$$
$$\varepsilon_2 = 2\varepsilon_1 + \frac{\ddot{H}}{H\dot{H}}.$$

- Duration of inflation $\varepsilon_i \ll 1$

$$N = \ln\frac{a_{end}}{a} = \int_{t_{cmb}}^{t_{end}} \mathrm{dln}a = \int_{t_{cmb}}^{t_{end}} H dt = \int_{\phi_{cmb}}^{\phi_{end}} \frac{H}{\dot{\phi}} d\phi$$

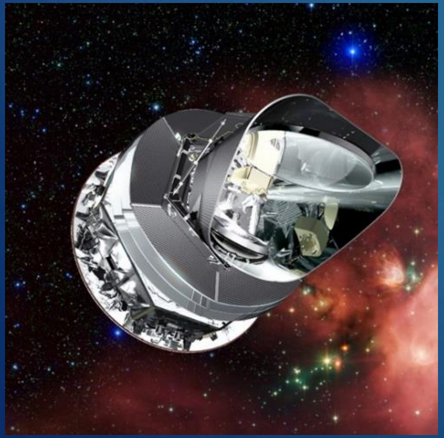
- The end of inflation $\epsilon_1(t_{end}) \approx 1$
- ***Independent observational parameters***: tensor-to-scalar ratio r and scalar spectral index n_s

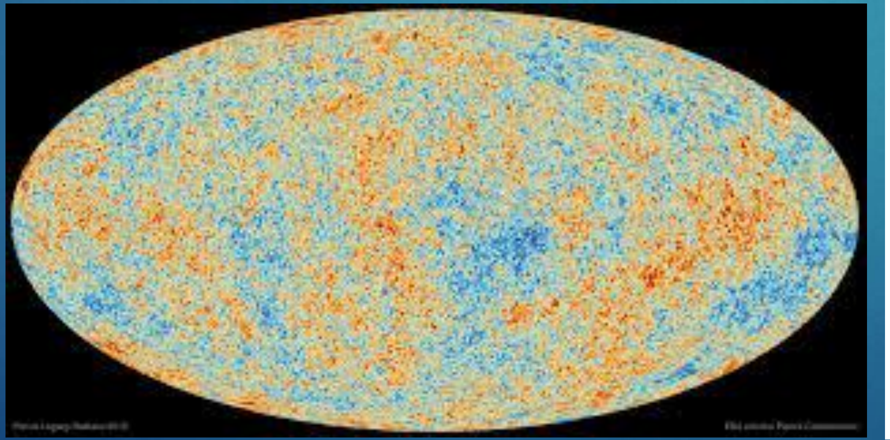
$$r = 16\varepsilon_1(t_{cmb})$$
$$n_s = 1 - 2\varepsilon_1(t_{cmb}) - \varepsilon_2(t_{cmb})$$

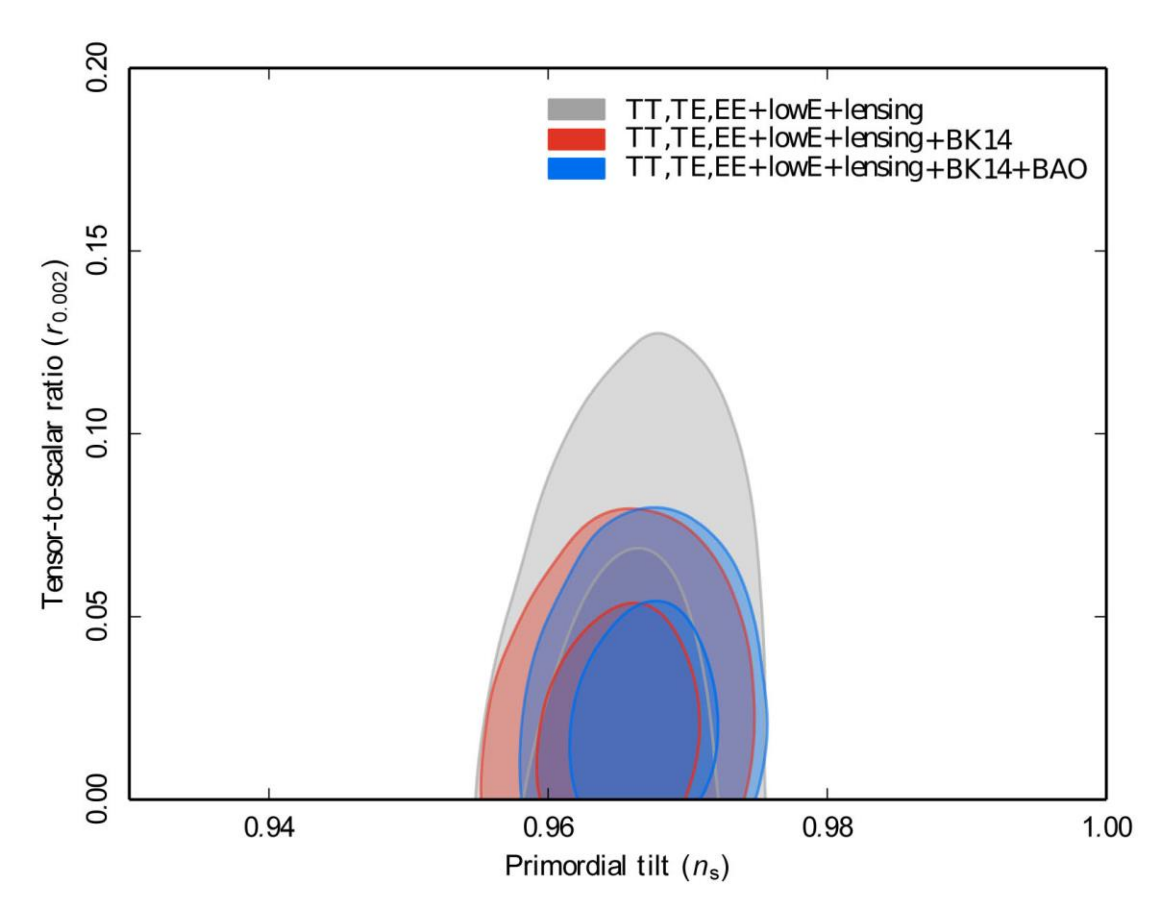
At the lowest order in parameters ε_1 and ε_2

Observational parameters

- ***Three independent observational parameters***: amplitude of scalar perturbation A_s, tensor-to-scalar ratio r and scalar spectral index n_s
- Satellite ***Planck*** (May 2009 – October 2013)
- ***Planck Collaboration***
  - Latest results are published in year 2018.







Planck 2018 results. X Constraints on inflation, arXiv:1807.06211 [astro-

Numerical simulations

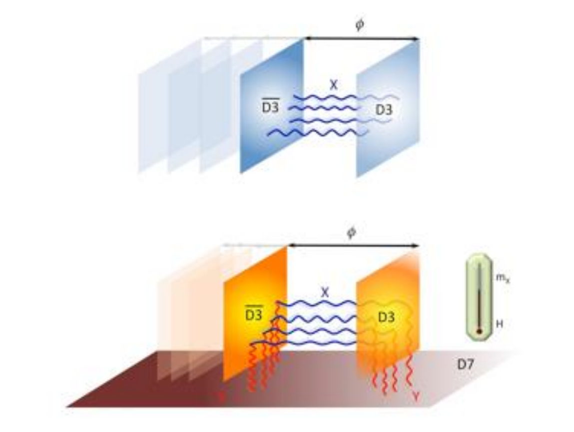
- For each model two potentials
  - $V(\theta) = e^{-\omega\cdot\theta}$, $\chi(\theta) = e^{\frac{1}{4}\omega\cdot\theta}$
  - $V(\theta) = \frac{1}{\cosh(\omega\cdot\theta)}$, $\chi(\theta) = \cosh^{\frac{1}{4}}(\omega\cdot\theta)$
- About 10000 simulations for different sets of
  - Free parameters N, κ, ω (random numbers in the given interval)
  - Initial conditions θ_i, π_i, h_i (random numbers and/or slow-roll approximation)
- System of Hamilton's equations is solved
  - $\epsilon_1, \epsilon_2, \epsilon_3$, etc. are calculated
  - t_{end} and θ_{end} are calculated from $\epsilon_1(t_{end}) = 1$
  - t_{cmb} is calculated for a given N, $N = \int_{t_{cmb}}^{t_{end}} H dt$
  - Observational parameters n_s and r, as a functions of $\epsilon_i(t_{cmb})$, $h(t_{cmb})$ and its derivatives are calculated

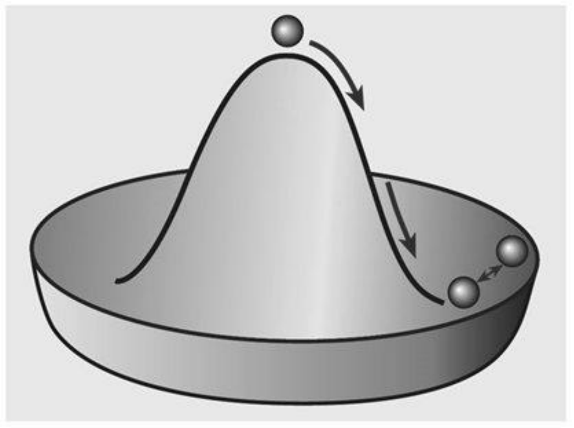
Models and approximations

- Models:
  - Tachyon inflation
  - Randall-Sundrum II extended (RSII + matter in the bulk)
  - Holographic (RSII) braneworld
- Approximation of (n_s, r):
  - for SSFI
  - for DBI inflation
  - *k*-inflation (high-order corrections)

Tachyons

- ***Traditionally***, the word tachyon was used to describe a ***hypothetical particle*** which propagates ***faster than light***.
- In modern physics this meaning has been changed:
  - The effective tachyonic field theory was **proposed** by ***A. Sen***
  - ***String theory***: states of quantum fields with imaginary mass (i.e., negative mass squared).
  - However, it **was realized** that the imaginary mass creates an instability and tachyons spontaneously decay through the process known as ***tachyon condensation.***
  - Quanta are not tachyon anymore, but rather an "ordinary" particle with a positive mass.





Lagrangian of a scalar field - $\mathcal{L}(X,\varphi)$

- In general case – any function of a scalar field φ and kinetic energy $X \equiv \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi$.
  - Canonical field with potential $V(\varphi)$

$$\mathcal{L}(X,\varphi) = BX - V(\varphi),$$

  - Non-canonical models

$$\mathcal{L}(X,\varphi) = BX^n - V(\varphi),$$

  - Dirac-Born-Infeld (DBI) Lagrangian

$$\mathcal{L}(X,\varphi) = -\frac{1}{f(\varphi)}\sqrt{1-2f(\varphi)X} - V(\varphi),$$

    - Special case – tachyonic $\mathcal{L}(X,\varphi) = -V(\varphi)\sqrt{1-2\lambda X},$

Tachyon inflation

- Properties of a tachyon potential

$$V(0) = const, \quad V'(\theta > 0) < 0, \quad V(|\theta| \to \infty) \to 0.$$

- The corresponding Lagrangian and the Hamiltonian are

$$p \equiv \mathcal{L}(\dot{\theta}, \theta) = -V(\theta)\sqrt{1 - \dot{\theta}^2}$$

the non-canonical (DBI) Lagrangian

$$\rho \equiv \mathcal{H} = \frac{V(\theta)}{\sqrt{1 - \dot{\theta}^2}}$$

- The Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\mathcal{H} = \frac{8\pi G}{3}\frac{V(\theta)}{\sqrt{1 - \dot{\theta}^2}}$$

D. Steer, F. Vernizzi, Tachyon inflation: *Tests and comparison with single scalar field inflation*, Phys. Rev. D. 70 (2004) 43527.

M. Milosevic, D.D. Dimitrijevic, G.S. Djordjevic, M.D. Stojanovic, *Dynamics of tachyon fields and inflation*, Serbian Astronomical Journal. 192 (2016)

Tachyon inflation

- Dynamic of inflation (nondimensional equations)

$$\dot{\theta} = \frac{\pi}{\sqrt{V^2 + \pi^2}}$$

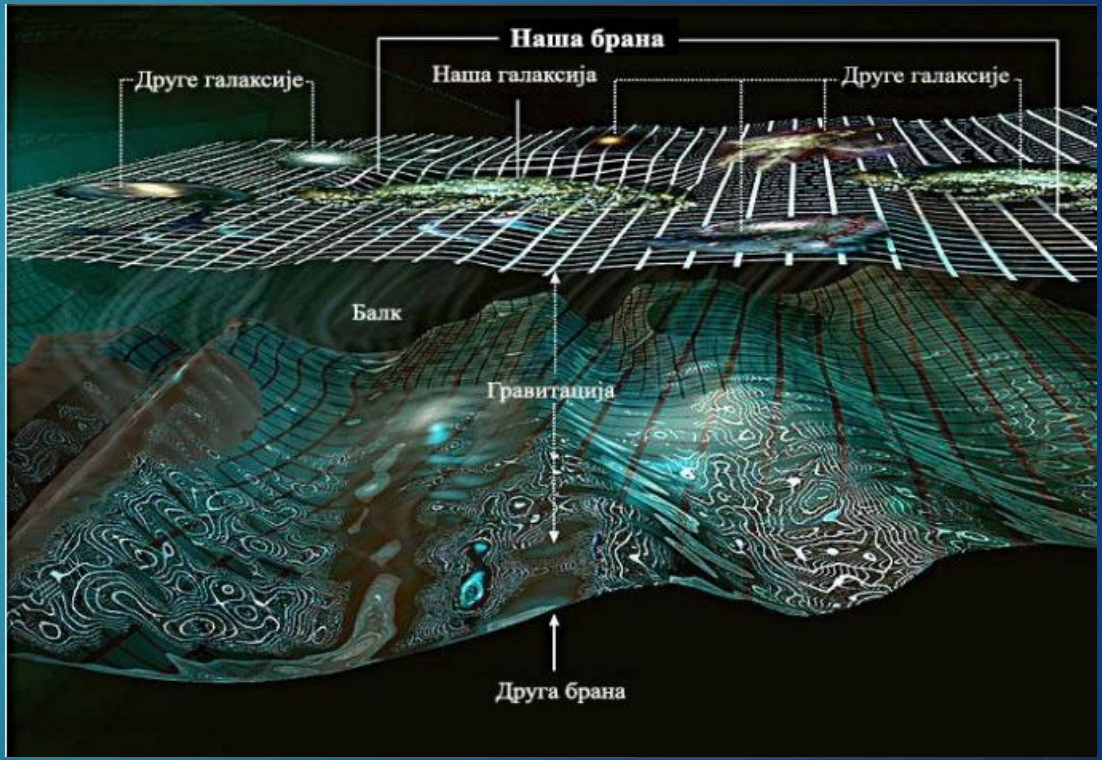
$$\dot{\pi} = -3h\pi - \frac{VV'}{\sqrt{V^2 + \pi^2}}$$

$$h^2 = \frac{\kappa^2}{3}\rho = \frac{\kappa^2}{3}\frac{V}{\sqrt{1 - \dot{\Theta}^2}}$$

- Dimensionless constant $\kappa^2 = 8\pi G\ell^{-2}$, where ℓ is the AdS curvature.
  - Rescaling $\tau = t/\ell$, $\theta = \Theta/\ell$, $h = \ell H$.

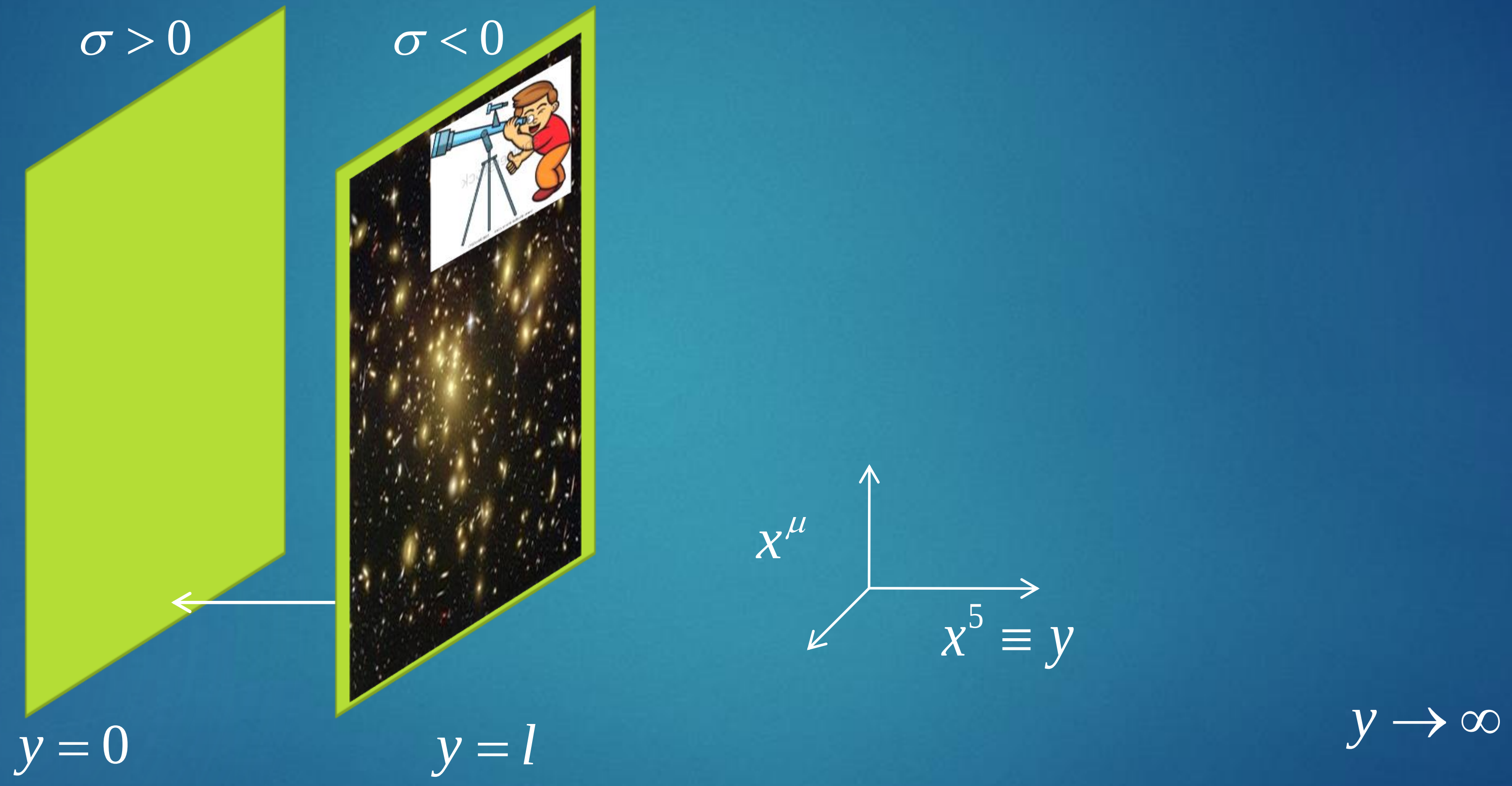
Braneworld cosmology

- Braneworld universe is based on the scenario
in which matter is confined on a brane moving in the higher dimensional bulk with only gravity allowed to propagate in the bulk.
- One of the simplest models - Randall-Sundrum (RS)
- RS model was originally proposed to solve the hierarchy problem (1999)
- Later it was realized that this model, as well as any similar braneworld model, may have interesting cosmological implications
- Two branes with opposite tensions are placed at some distance in 5 dimensional space



The RS Model

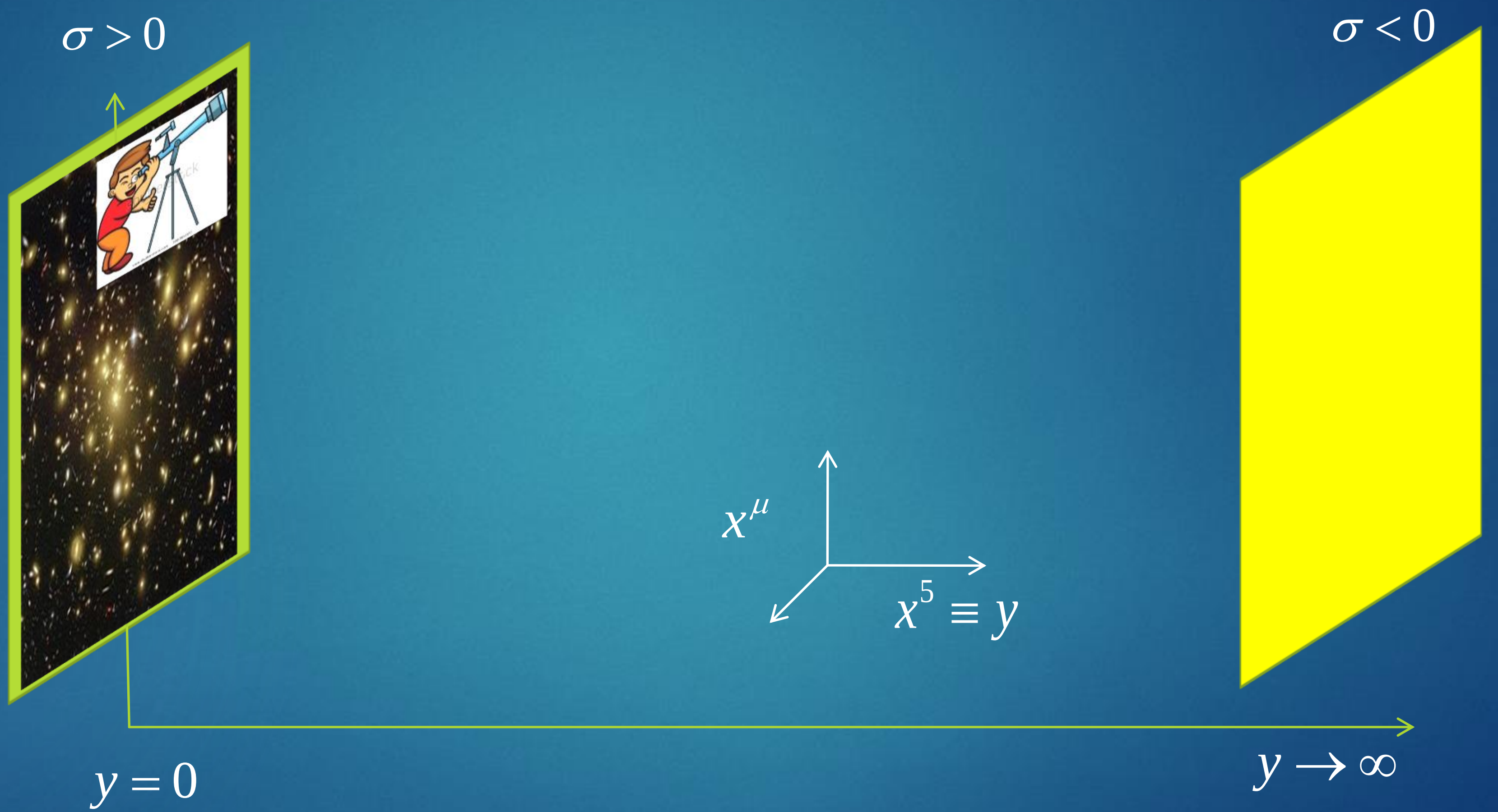
- Observer reside on the brane with negative tension,
- Distance to the 2nd brane corresponds to the Netwonian gravitational constant



Animation: N. Bilic, “Braneworld Universe”, 2nd CERN – SEENET-MTP PhD School, Timisoara, Dece

The RSII Model

- Observer is placed on the positive tension brane
- 2^{nd} brane is pushed to infinity



Animation: N. Bilic, “Braneworld Universe”, 2nd CERN – SEENET-MTP PhD School, Timisoara, December 2016

The (extended) RSII Model

- The **original RSII** model consists of two D3-branes in the 4 + 1 dimensional anti-de Sitter (AdS_5) background with line element

$$ds^2_{(5)} = e^{-2k/\ell} g^{\mu\nu} dx^\mu dx^\nu - dy^2$$

with the observer brane placed at $y = 0$ and the negative tension brane pushed of to $y \to \infty$.

- One additional brane dynamical 3-brane moving in the AdS_5 bulk behaves effectively a tachyon with a potential $V(\theta) \propto \theta^{-4}$.
- The **extended RSII** model - the RSII model is extended to include matter in the bulk.
- The presence of matter modifies the warp factor which results in two effects:
  - a modification of the RSII cosmology
  - a modification of the tachyon potential.

N. Bilic, G.B. Tupper, *AdS braneworld with backreaction*, Cent. Eur. J. Phys. 12 (2014) 147–159.

N. Bilic, D.D. Dimitrijevic, G.S. Djordjevic, M. Milosevic, *Tachyon inflation in an AdS braneworld with back-reaction*, IJMPA.

The (extended) RSII Model

- The corresponding Lagrangian and the Hamiltonian are

$$p = \frac{\mathcal{L}}{\sigma} = -\frac{1}{\chi^4\sqrt{1+\chi^8\pi_\theta^2}} = -\frac{1}{\chi^4}\sqrt{1-\dot{\theta}^2}$$

$$\rho = \frac{\mathcal{H}}{\sigma} = \frac{1}{\chi^4}\sqrt{1+\chi^8\pi_\theta^2} = \frac{1}{\chi^4}\cdot\frac{1}{\sqrt{1-\dot{\theta}^2}}$$

where π_θ is the conjugate momentum, i.e. $\pi_\theta = \frac{\partial\mathcal{L}}{\partial\theta}$.

- Tachyonic potential is given as

$$V(\theta) = \sigma\chi^{-4}(\theta),$$

where $\chi(\theta)$ is arbitrary function depends on the self-interaction potential of the bulk scalar field.

N. Bilić, S. Domazet, G.S. Djordjevic, *Particle Creation and Reheating in a Braneworld Inflationary Scenario*, Physical Review D, 96 (2017), 083518

N. Bilić, S. Domazet, and G. S. Djordjevic, *Tachyon with an inverse power-law potential in a braneworld cosmology*, Class. Quantum Gravity 34,

The (extended) RSII Model

$$\chi_{,\theta} = \frac{\partial \chi}{\partial \theta}$$

- Dynamic of inflation (nondimensional equations)

$$\dot{\theta} = \frac{\chi^4 \pi_\theta}{\sqrt{1 + \chi^8 \pi_\theta^2}} = \frac{\pi_\theta}{\rho}$$

$$\dot{\pi}_\theta = -3h\pi_\theta + \frac{4\chi_{,\theta}}{\chi^5 \sqrt{1 + \chi^8 \pi_\theta^2}}$$

- The modified Friedmann equations

$$h^2 = \frac{\kappa^2}{3}\rho\left(\chi_{,\varphi} + \frac{\kappa^2}{12}\rho\right)$$

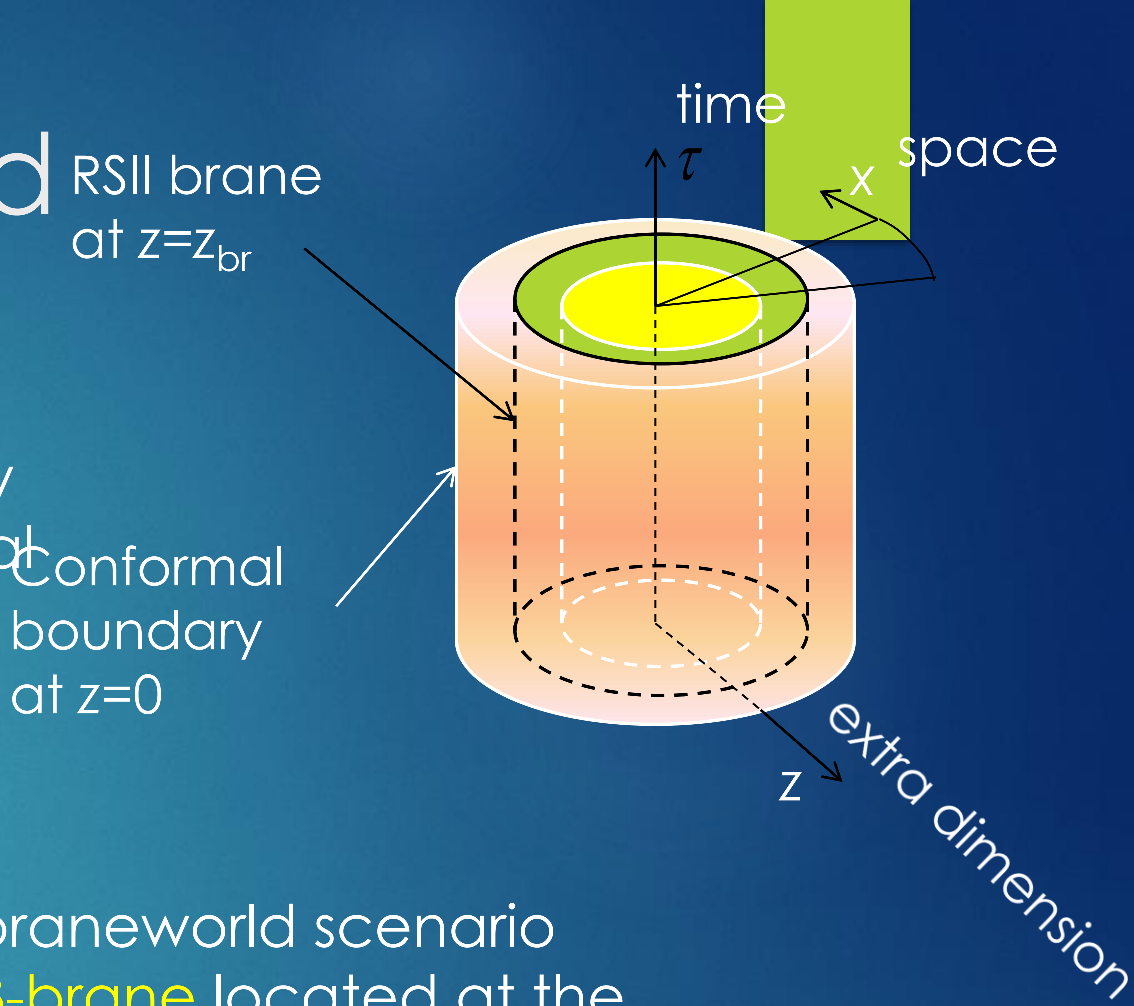
$$\dot{h} = -\frac{\kappa^2}{2}(\rho + p)\left(\chi_{,\theta} + \frac{\kappa^2}{6}\rho\right) + \frac{\kappa^2 \rho}{6h}\chi_{,\theta\theta}\dot{\theta}$$

$$h^2 = \frac{\kappa^2}{3}\rho \qquad h^2 = \frac{\kappa^2}{3}\rho\left(1 + \frac{\kappa^2}{12}\rho\right)$$

$$\dot{h} = -\frac{\kappa^2}{2}(p + \rho) \qquad \dot{h} = -\frac{\kappa^2}{2}(p + \rho)\left(1 + \frac{\kappa^2}{6}\rho\right)$$

- The dimensionless constant $\kappa^2 = 8\pi\sigma G\ell^{-2} = 8\pi\sigma G_5 G^{-1}$; where σ is brane tension, ℓ is the AdS curvature, and the mass scale $\frac{1}{\ell} = \frac{G}{G_5}$ is fixed from phenomenology.
  - Rescaling $\tau = t/\ell$, $\theta = \Theta/\ell$, $\pi_\theta = \Pi_\Theta/\sigma$ and $h = \ell H$

Holographic braneworld



- Holographic braneworld - a cosmology based on the effective four-dimensional Einstein equations on the holographic boundary in the framework of anti de Sitter/conformal field theory (AdS/CFT) correspondence.
- The model is based on a holographic braneworld scenario with an effective tachyon field on a D3-brane located at the holographic boundary of an asymptotic AdS_5 bulk.
- The cosmology is governed by matter on the brane in addition to the boundary CFT

Holographic tachyon cosmology

- The holographic braneworld is a spatially flat FRW universe with line element
$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2)$$
- The holographic Friedmann equations
$$h^2 - \frac{\ell^2}{4}h^4 = \frac{\kappa^2}{3}\ell^4\rho$$
$$\dot{h}\left(1 - \frac{\ell^2}{2}h^2\right) = -\frac{\kappa^2}{3}\ell^3(p+\rho)$$
- Where the scale ℓ can be identified with the AdS curvature radius and we introduced a dimensionless expansion rate $h \equiv \ell H$ and the fundamental dimensionless coupling
$$\kappa^2 = \frac{8\pi G}{\ell^2}$$

Standard cosmology:
$$h^2 = \frac{\kappa^2}{3}\rho$$
$$\dot{h} = -\frac{\kappa^2}{2}(p+\rho)$$

Extended RSII cosmology
$$h^2 = \frac{\kappa^2}{3}\rho\left(\chi_{,\varphi} + \frac{\kappa^2}{12}\rho\right)$$
$$\dot{h} = -\frac{\kappa^2}{2}(\rho+p)\left(\chi_{,\theta} + \frac{\kappa^2}{6}\rho\right) + \frac{\kappa^2\rho}{6h}\chi_{,\theta\theta}\dot{\theta}$$

Bilić, N., Dimitrijević, D. D., Djordjevic, G. S., Milošević, M. & Stojanović, M. *Tachyon inflation in the holographic braneworld*. JCAP **2019**, 034–034 (2019).

Holographic tachyon cosmology

- Interesting property - solving the first Friedmann equation as a quadratic equation

$$h^2 = 2\left(1 \pm \sqrt{1 - \frac{\kappa^2}{3}\ell^4\rho}\right)$$

  - We do not want our modified cosmology to depart too much from the standard cosmology after the inflation era and demand that this equation reduces to the standard Friedmann equation in the low density limit $(\kappa^2\ell^4\rho \ll 1)$
  - This demand will be met only by the (−) sign solution. We discard the (+) sign solution as unphysical.
- The physical range of the Hubble expansion rate is between $h_{\min} = 0$ and the maximal value $h_{\max} = \sqrt{2}$
  - It corresponds to the maximal energy density $\rho_{\max} = 3/(\kappa^2\ell^4)$
- Assuming no violation of the weak energy condition $p + \rho \geq 0$, the expansion rate will be a monotonously decreasing function of time.
- The universe starts from $t = 0$ with an initial $h_i \leq h_{max}$ with energy density and cosmological scale both finite.
- The Big Bang singularity is avoided.

Holographic tachyon cosmology

- The nondimensional equations of motions

$$\dot{\theta} = \frac{\eta}{\sqrt{1+\eta^2}}$$

$$\dot{\eta} = -\frac{3h\eta}{\ell} - \frac{V_{,\theta}}{V}\left(\sqrt{1+\eta^2} + \frac{\eta^2}{\sqrt{1+\eta^2}}\right)$$

where $\eta = \frac{\sqrt{g_{\mu\nu}\pi^\mu\pi^\nu}}{\ell^4 V}$

- As usual, the pressure and energy density are equal to Lagrangian and Hamiltonian

$$p \equiv \mathcal{L} = -\ell^{-4}V\sqrt{1-\dot{\theta}^2} = -\frac{\ell^{-4}V}{\sqrt{1-\eta^2}}$$

$$\rho \equiv \mathcal{H} = \frac{\ell^{-4}V}{\sqrt{1-\dot{\theta}^2}} = \ell^{-4}V\sqrt{1-\eta^2}$$

Summary of the Models

SSFI	Tachyon	RSII Extended	Holographic
$\dot{\theta} = \pi$ $\dot{\pi} = -3h\pi - V'$	$\dot{\theta} = \frac{\pi}{\sqrt{V^2 + \pi^2}}$ $\dot{\pi} = -3h\pi - \frac{VV'}{\sqrt{V^2 + \pi^2}}$	$\dot{\theta} = \frac{\chi^4 \pi_\theta}{\sqrt{1 + \chi^8 \pi_\theta^2}} = \frac{\pi_\theta}{\rho}$ $\dot{\pi}_\theta = -3h\pi_\theta + \frac{4\chi_{,\theta}}{\chi^5 \sqrt{1 + \chi^8 \pi_\theta^2}}$	$\dot{\theta} = \frac{\eta}{\sqrt{1 + \eta^2}}$ $\dot{\eta} = -\frac{3h\eta}{\ell} - \frac{V_{,\theta}}{V}\left(\sqrt{1 + \eta^2} + \frac{\eta^2}{\sqrt{1 + \eta^2}}\right)$
$h^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\dot{\theta}^2 + V\right)$	$h^2 = \frac{\kappa^2}{3}\frac{V}{\sqrt{1 - \dot{\theta}^2}}$	$h^2 = \frac{\kappa^2}{3}\rho\left(\chi_{,\varphi} + \frac{\kappa^2}{12}\rho\right)$	$h^2 = 2\left(1 \pm \sqrt{1 - \frac{\kappa^2}{3}\rho}\right)$

mainModel class

The child class for each model:
Inherits everything that is common, override what is necessary

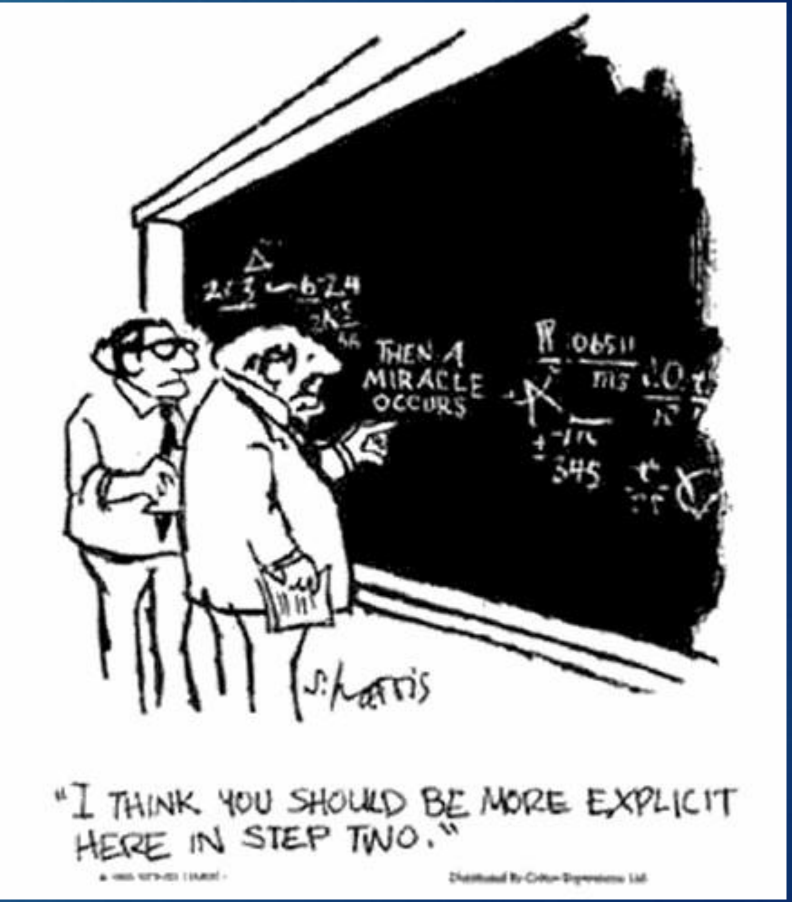
Observational parameters (n_s, r)

- SSFI, Tachyon and RSII Extended inflation
- The second order of parameters ε_{i}

$$r = 16\varepsilon_1(1 + C\varepsilon_2 - 2\alpha\varepsilon_1)$$

$$n_s = 1 - 2\varepsilon_1 - \varepsilon_2 - [2\varepsilon_1^2 + (2C + 3 - 2\alpha)\varepsilon_1\varepsilon_2 + C\varepsilon_2\varepsilon_3]$$

- In all models we will discuss here $C \simeq -0{,}72$, however:
  - $\alpha = 0$ for SSFI,
  - $\alpha = \frac{1}{6}$ for tachyon inflation in standard cosmology,
  - $\alpha = \frac{1}{12}$ for Randall-Sundrum cosmology.



Observational parameters (n_s, r)

- Holographic cosmology

$$r = 16\varepsilon_1\left[1 + C\varepsilon_2 + \frac{2(2-h^2)}{3(4-h^2)}\frac{pp_{,XX}}{p_{,X}^2}\varepsilon_1\right]$$

$$n_S = 1 - 2\varepsilon_1 - \varepsilon_2 - \left(2 + \frac{8h^2}{3(4-h^2)^2}\frac{pp_{,XX}}{p_{,X}^2}\right)\varepsilon_1^2 - \left(3 + 2C + \frac{2(2-h^2)}{3(4-h^2)}\frac{pp_{,XX}}{p_{,X}^2}\right)\varepsilon_1\varepsilon_2 - C\varepsilon_2\varepsilon_3$$

- For $X = \dot{\theta}^2$ and $p = -V\sqrt{1-X}$ we have $\frac{pp_{,XX}}{p_{,X}^2} = -1$

Bertini, N. R., Bilic, N. & Rodrigues, D. C. Primordial perturbations and inflation in holographic cosmology. arXiv:2007.02332 [gr-qc]

Case 1: Approximation of (n_s, r)

- Parametrizing the spectra, for example by power-laws, is well suited to testing the infationary models but will only correctly estimate cosmological parameters if the parametrization is suffciently accurate.
- There are different approaches to approximately calculate and parametrise power spectra.
  - all of these approaches have some advantages or disadvantages but leads to the same final expressions
- The scalar and tensor spectral index at *n*-th order an be written as

$$n_s - 1 \approx -2(\epsilon_1 + \epsilon_1^2 + \cdots + \epsilon_1^n) - \epsilon_2$$
$$n_T \approx -2(\epsilon_1 + \epsilon_1^2 + \cdots + \epsilon_1^n)$$

$$n_T = -\frac{r}{8} \quad \rightarrow \quad n_T = -2\left[\left(\frac{r}{16}\right) + \left(\frac{r}{16}\right)^2 + \cdots + \left(\frac{r}{16}\right)^n\right]$$

- Works for Standard Single Field Inflation

E. D. Stewart, D. H. Lyth, *A more accurate analytic calculation of the spectrum of cosmological perturbations produced during inflation*. Physics Letters B 302, 171–175 (1993)

S. M. Leach, et al. *Cosmological parameter estimation and the inflation cosmology*. Physical Review D, 66(2):023515 (2002)

D. J. Schwarz, et al. *Higher order corrections to primordial spectra from cosmological inflation*. Physics Letters B, 517(3-4):243249 (2001)

Case 2: Approximation of (n_s, r)

- One of the main predictions of inflationary DBI models is that the inflaton perturbations can propagate with a sound speed $c_s < 1$
- Sound speed is defined as

$$c_s^2 = \left.\frac{\partial p}{\partial \rho}\right|_\theta = \frac{p_{,X}}{\rho_{,X}}$$

- At leading order in the slow-roll parameters the scalar power spectrum depends on the sound speed

$$n_s - 1 = -2\epsilon_1 - \epsilon_2 - s$$
$$r \approx 16\epsilon_1 c_s$$

where $s = \frac{\dot{c}_s}{c_s H}$.

Tachyon / RSII models

$$c_s = 1 - 2\alpha\epsilon_1$$

Holography

$$c_s^2 = 1 - \frac{4(2-h^2)}{3(4-h^2)}\epsilon_1$$

J. Garriga, V. F. Mukhanov, *Perturbations in k-inflation*. Physics Letters B, 458, 219 (2019)
Planck Collaboration *et al.* Planck 2015 results. XX. Constraints on inflation. *Astronomy and Astrophysics* **594**, A20 (2015)

Case 3: Approximation of (n_s, r)

- This approximation is valid for k-inflation
- It represents the most general SSFI, in which the perturbations usually obey an equation of motion with a time-dependent sound speed
- Based on high-order uniform asymptotic approximation method
  - the slow-roll expressions of the parameters (n_s, r) are written in terms of the Hubble and sound speed flow parameters
- Two conditions that k-inflation must satisfy:
  - $\frac{\partial P}{\partial X} > 0$ and $2X\frac{\partial^2 P}{\partial X^2} + \frac{\partial P}{\partial X} > 0$
- Sound speed $c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$
  - Hierarchy of sound speed $q_{n+1} = \frac{d \ln q_n}{d \ln a}$, $q_0 = \frac{c_*}{c_s}$

Zhu, T., Wang, A., Cleaver, G., Kirsten, K. & Sheng, Q. Power spectra and spectral indices of *k*-inflation: high-order corrections. *Phys. Rev. D* **90**, 103517 (2014).

Case 3: Aproximation of (n_s, r)

$$
\begin{aligned}
n_s =\ & 1 + q_1 - 2\epsilon_1 - \epsilon_2 - q_1^2 + \left(\frac{64}{27} - \ln\frac{3}{c_0}\right) q_1 q_2 + 3 q_1 \epsilon_1 - 2\epsilon_1^2 + q_1 \epsilon_2 + \\
& + \left(-\frac{101}{27} + 2\ln\frac{3}{c_0}\right)\epsilon_1\epsilon_2 + \left(-\frac{10}{27} + \ln\frac{3}{c_0}\right)\epsilon_2\epsilon_3 + \\
& + \left(\frac{73}{81} - \ln\frac{3}{c_0}\right) q_1 q_2 \epsilon_2 + q_1^3 - 4q_1^2\epsilon_1 + 5q_1\epsilon_1^2 - 2\epsilon_1^3 - q_1^2\epsilon_2 + \\
& + \frac{38}{81}\epsilon_2^2\epsilon_3 + \left(-\frac{442}{81} - \frac{2867\ln 2}{315} + \frac{9\ln 4}{2} + 3\ln\frac{3}{c_0}\right) q_1^2 q_2 + \left(\frac{803}{81} - 5\ln\frac{3}{c_0}\right) q_1\epsilon_1\epsilon_2 + \\
& + \left(\frac{19}{324} + \frac{\pi^2}{24} + \frac{10}{27}\ln\frac{3}{c_0} - \frac{1}{2}\ln^2\frac{3}{c_0}\right)\epsilon_2\epsilon_3^2 + \left(\frac{260}{81} - \frac{\pi^2}{24} - \frac{64}{27}\ln\frac{3}{c_0} + \frac{1}{2}\ln^2\frac{3}{c_0}\right) q_1 q_2^2 + \\
& + \left(\frac{260}{81} - \frac{\pi^2}{24} - \frac{64}{27}\ln\frac{3}{c_0} + \frac{1}{2}\ln^2\frac{3}{c_0}\right) q_1 q_2 q_3 + \left(\frac{611}{81} - 4\ln\frac{3}{c_0}\right) q_1 q_2 \epsilon_1 + \\
& + \left(\frac{19}{324} + \frac{\pi^2}{24} + \frac{10}{27}\ln\frac{3}{c_0} - \frac{1}{2}\ln^2\frac{3}{c_0}\right)\epsilon_2\epsilon_3\epsilon_4 + \left(-\frac{55}{18} + \frac{\pi^2}{12} + \frac{101}{27}\ln\frac{3}{c_0} - \ln^2\frac{3}{c_0}\right)\epsilon_1\epsilon_2^2 + \\
& + \left(-\frac{757}{81} + 6\ln\frac{3}{c_0}\right)\epsilon_1^2\epsilon_2 + \left(\frac{103}{81} - 2\ln\frac{3}{c_0}\right) q_1\epsilon_2\epsilon_3 + \\
& + \left(-\frac{185}{54} + \frac{\pi^2}{12} + \frac{128}{27}\ln\frac{3}{c_0} - \ln^2\frac{3}{c_0}\right)\epsilon_1\epsilon_2\epsilon_3
\end{aligned}
$$

$$
\begin{aligned}
r =\ & 16c_0\epsilon_1 + 16c_0\left(-\frac{429}{181} + \ln\frac{3}{c_0}\right)\epsilon_1 q_1 + 32c_0\ln c_0\left(\epsilon_1\right)^2 + 16c_0\left(\frac{67}{181} - \ln\frac{3}{c_0}\right)\epsilon_1\epsilon_2 + \\
& + 16c_0\left(\frac{285365}{65522} + \frac{64\ln 2}{1267} - \frac{610}{181}\ln\frac{3}{c_0} + \frac{1}{2}\ln^2\frac{3}{c_0}\right) q_1^2\epsilon_1 + \\
& + 16c_0\left(-\frac{4865}{1629} + \frac{\pi^2}{24} + \frac{429}{181}\ln\frac{3}{c_0} - \frac{1}{2}\ln^2\frac{3}{c_0}\right) q_1 q_2 \epsilon_1 + 32c_0\left(\ln c_0 + \ln^2 c_0\right)\epsilon_1^3 + \\
& + 16c_0\left(4\ln 3 + \left(\frac{1401}{181} + \ln 9\right)\ln\frac{3}{c_0} - 2\ln^2\frac{3}{c_0} - \frac{508394}{98283} - \frac{1401\ln 3}{181}\right) q_1\epsilon_1^2 + \\
& + 16c_0\left(\frac{42500}{98283} + \frac{630\ln 3}{181} - \ln^2 3 - \left(\frac{811}{181} + \ln 9\right)\ln\frac{3}{c_0} + 3\ln^2\frac{3}{c_0}\right)\epsilon_1^2\epsilon_2 + \\
& + 16c_0\left(\frac{1}{2}\ln^2\frac{3}{c_0} - \frac{32615}{196566} - \frac{67}{181}\ln\frac{3}{c_0}\right)\epsilon_1\epsilon_2^2 + \\
& + 16c_0\left(-\frac{174811}{98283} + \frac{677}{181}\ln\frac{3}{c_0} - \ln^2\frac{3}{c_0}\right)\epsilon_1\epsilon_2 q_1 + \\
& + 16c_0\left(\frac{86}{1629} - \frac{\pi^2}{24} - \frac{67}{181}\ln\frac{3}{c_0} + \frac{1}{2}\ln^2\frac{3}{c_0}\right)\epsilon_1\epsilon_2\epsilon_3
\end{aligned}
$$

$c_s^2 < 1$

$c_0 = c_s(t_{CMB})$

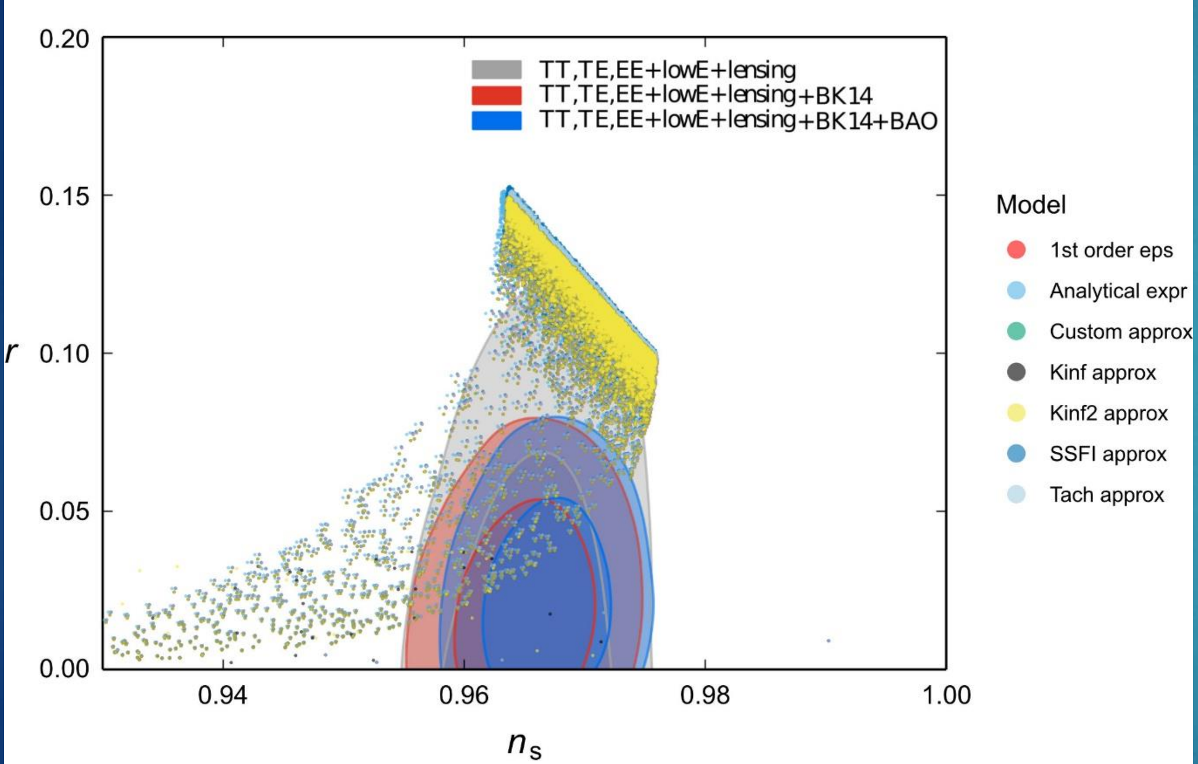
Zhu, T., Wang, A., Cleaver, G., Kirsten, K. & Sheng, Q. Power spectra and spectral indices of *k*-inflation: high-order corrections. *Phys. Rev. D* **90**, 103517 (2014).

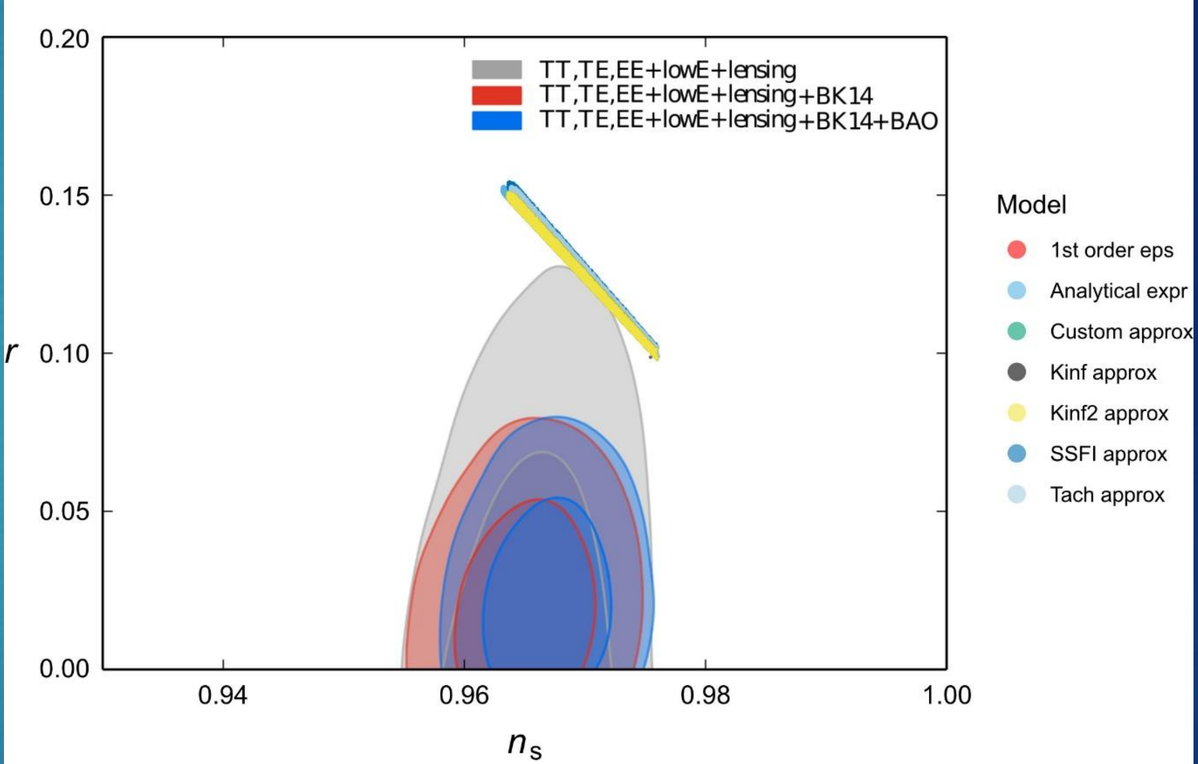
Case 3: Approximation of (n_s, r)

- Case 3a
  - Using only general expressions; numerical calculations, spline interpolations
    - ϵ_i, q_i, $\dot{c}_s$ (and higher derivatives)
- Case 3b - only for „holographic“ model!
  - Analytical expressions for
    - ϵ_1, ϵ_2, ϵ_3, ϵ_4
    - q_1, q_2, q_3
    - c_s, $\dot{c}_s$, $\ddot{c}_s$, $\dddot{c}_s$

- Numerical calculation (always)
  - $\dddot{h}$ and higher derivatives
- Analytical expressions (always)
  - c_s, $P \equiv \mathcal{L}$, $P_{,X}$, $P_{,XX}$

Tachyon inflation





$V(\theta) = 1/\cosh(\omega \cdot \theta)$

$V(\theta) = e^{-\omega \cdot \theta}$

$60 < N < 90, \; 0 < \omega < 1$

Relative distance $D_{rel} = \sqrt{\left(\frac{n_s^x - n_s}{n_s}\right)^2 + \left(\frac{r^x - r}{r}\right)^2}$

Tachyon inflation

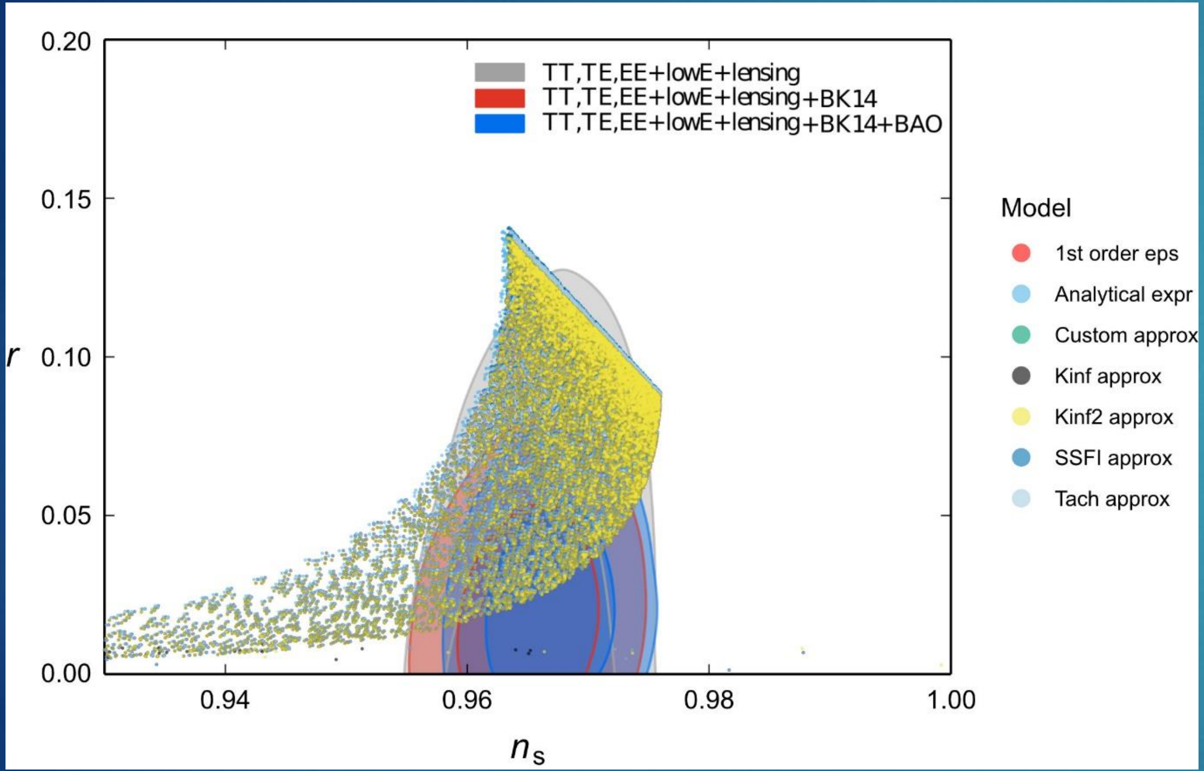
cosh FLRW cosm.	Mean	SD	Median	Min	Max
1st order	3.680E-04	1.148E-04	3.616E-04	4.002E-05	6.004E-04
Case 1	5.838E-03	1.990E-03	6.240E-03	1.918E-04	8.483E-03
Case 2	3.680E-04	1.148E-04	3.616E-04	4.002E-05	6.004E-04
Case 3a 3rd order	1.467E-02	9.273E-03	1.132E-02	8.435E-03	5.018E-02
Case 3a 2nd order	1.460E-02	9.315E-03	1.124E-02	8.376E-03	5.035E-02
Case 3b	/	/	/	/	/

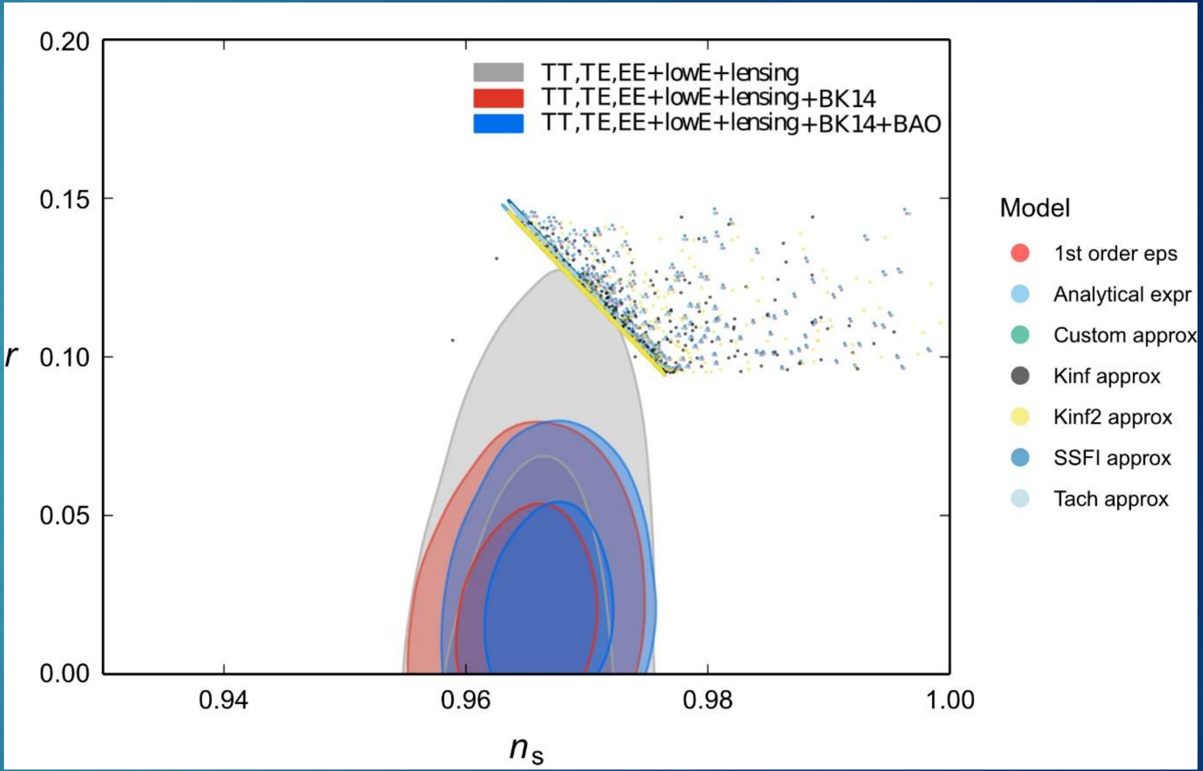
exp FLRW cosm.	Mean	SD	Median	Min	Max
1st order	3.882E-04	9.341E-05	3.696E-04	2.595E-04	5.948E-04
Case 1	6.885E-03	8.170E-04	6.766E-03	5.673E-03	8.575E-03
Case 2	3.882E-04	9.341E-05	3.696E-04	2.595E-04	5.948E-04
Case 3a 3rd order	1.006E-02	1.177E-03	9.891E-03	8.252E-03	1.255E-02
Case 3a 2nd order	9.970E-03	1.156E-03	9.809E-03	8.194E-03	1.242E-02
Case 3b	/	/	/	/	/

cosh RSII cosm.	Mean	SD	Median	Min	Max
1st order	4.359E-04	1.211E-04	4.183E-04	1.484E-05	6.944E-04
Case 1	6.925E-03	1.669E-03	7.056E-03	1.159E-04	9.573E-03
Case 2	4.359E-04	1.211E-04	4.183E-04	1.484E-05	6.944E-04
Case 3a 3rd order	1.249E-02	5.720E-03	1.114E-02	8.480E-03	5.928E-02
Case 3a 2nd order	1.243E-02	6.064E-03	1.105E-02	8.416E-03	1.563E-01
Case 3b	/	/	/	/	/

exp RSII cosm.	Mean	SD	Median	Min	Max
1st order	4.583E-04	1.113E-04	4.396E-04	2.979E-04	6.972E-04
Case 1	7.734E-03	9.465E-04	7.635E-03	6.225E-03	9.667E-03
Case 2	4.583E-04	1.113E-04	4.396E-04	2.979E-04	6.972E-04
Case 3a 3rd order	1.028E-02	1.207E-03	1.014E-02	8.249E-03	1.291E-02
Case 3a 2nd order	1.018E-02	1.184E-03	1.005E-02	8.185E-03	1.277E-02
Case 3b	/	/	/	/	/

The (extended) RSII model





$V(\theta) = 1/\cosh(\omega \cdot \theta)$

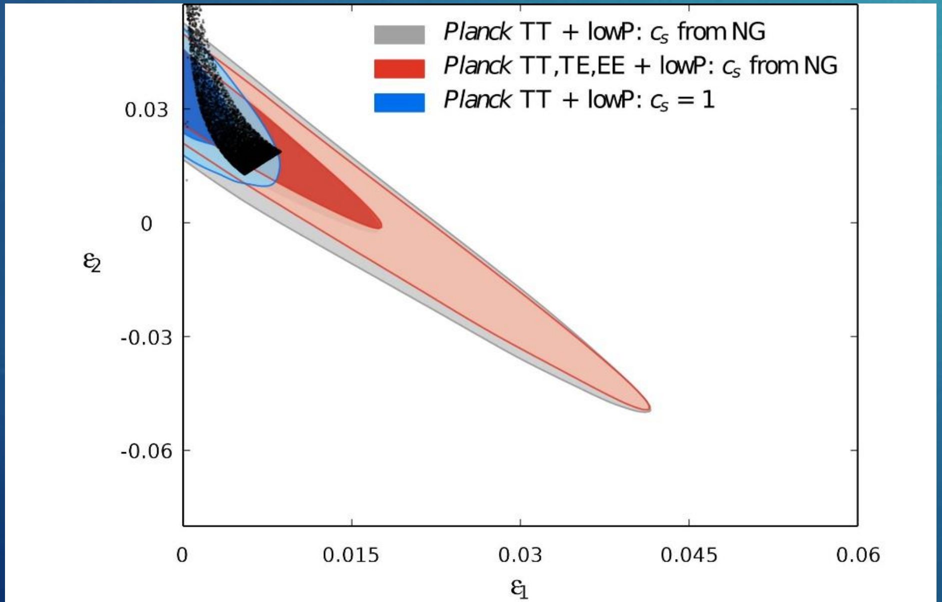
$V(\theta) = e^{-\omega \cdot \theta}$

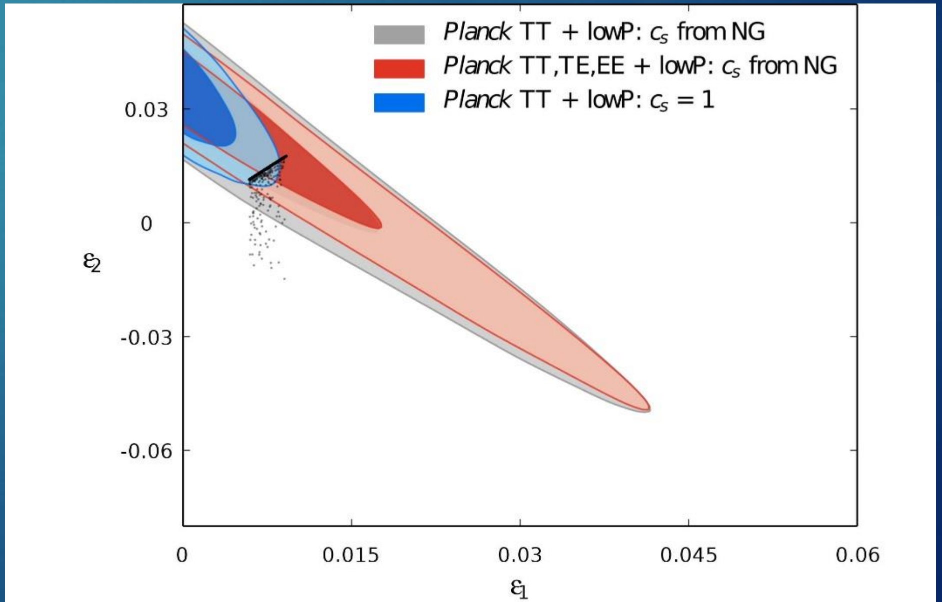
$60 < N < 90, \; 0 < \omega < 1$

The (extended) RSII model

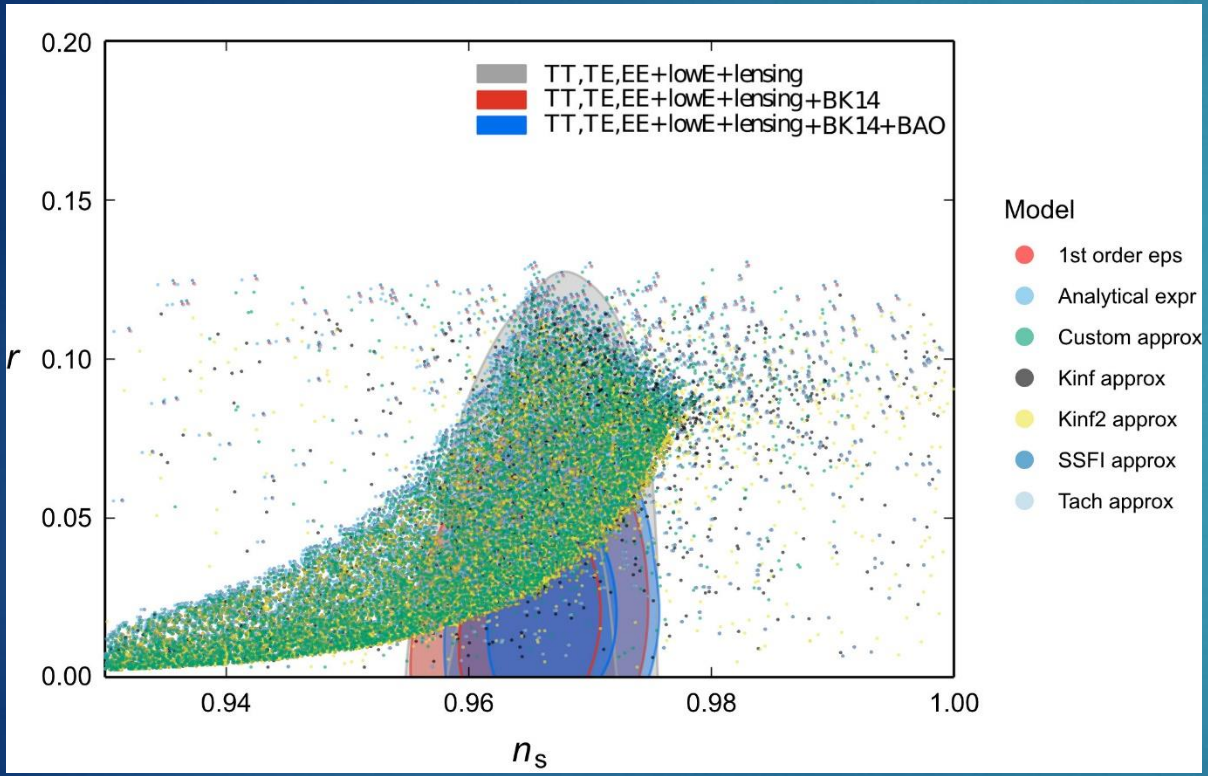
cosh	Mean	SD	Median	Min	Max
1st order	3.639E-04	1.249E-04	3.493E-04	3.011E-06	6.743E-04
Case 1	4.962E-03	1.981E-03	5.314E-03	5.400E-05	8.825E-03
Case 2	3.639E-04	1.249E-04	3.493E-04	3.011E-06	6.743E-04
Case 3a 3rd order	1.771E-02	9.395E-03	1.493E-02	9.803E-03	5.716E-01
Case 3a 2nd order	1.774E-02	1.378E-02	1.482E-02	9.739E-03	1.116E+00
Case 3b	/	/	/	/	/

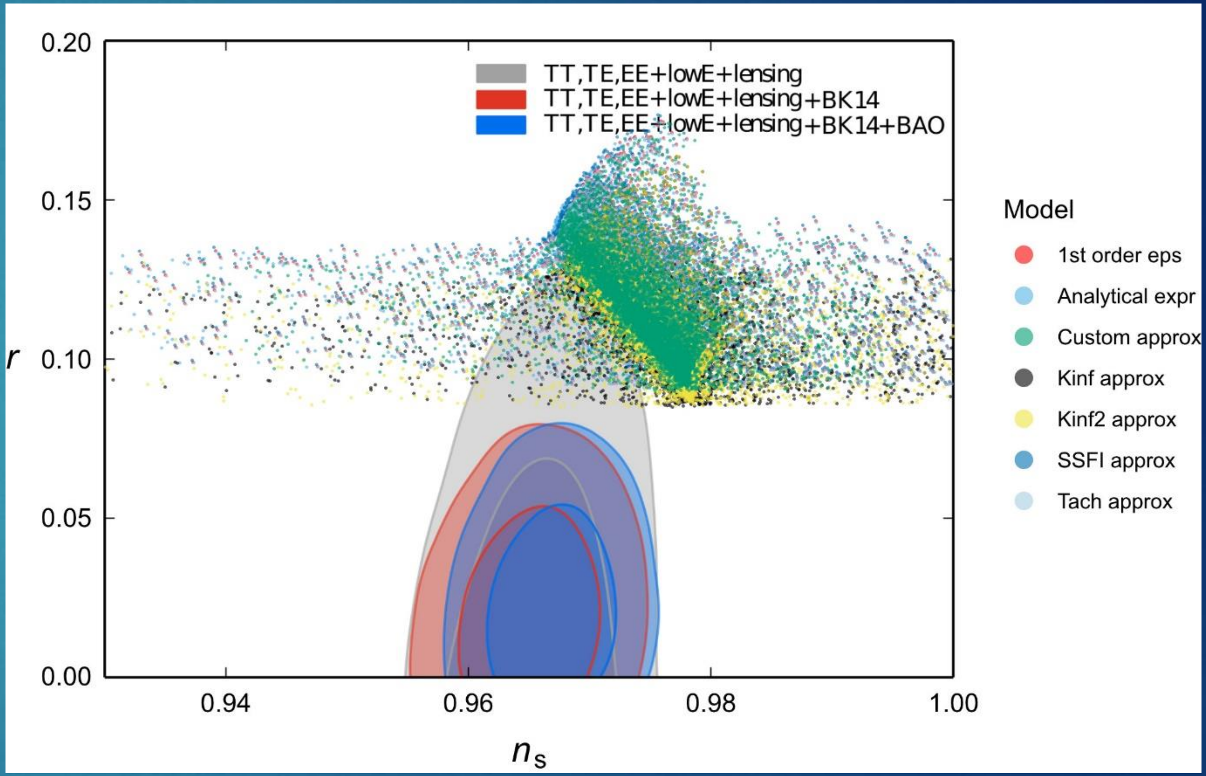
exp	Mean	SD	Median	Min	Max
1st order	4.505E-04	1.581E-04	4.350E-04	3.662E-08	1.139E-02
Case 1	7.502E-03	9.323E-04	7.428E-03	5.989E-03	1.874E-02
Case 2	4.504E-04	1.499E-04	4.350E-04	3.662E-08	1.017E-02
Case 3a 3rd order	1.144E-02	1.681E-03	1.131E-02	4.308E-04	6.771E-02
Case 3a 2nd order	1.158E-02	5.220E-03	1.124E-02	5.237E-04	2.767E-01
Case 3b	/	/	/	/	/





Holographic tachyon cosmology





$V(\theta) = 1/\cosh(\omega \cdot \theta)$

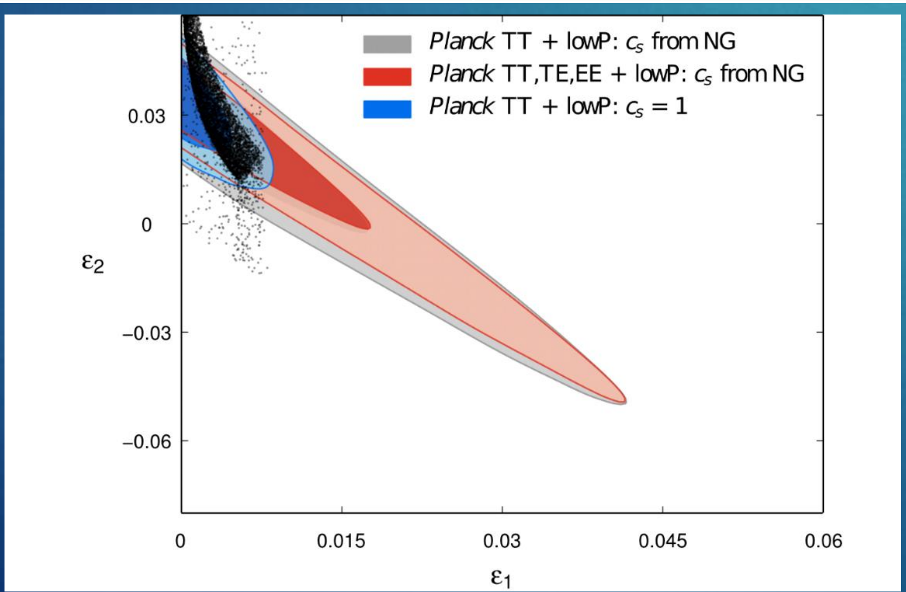
$V(\theta) = e^{-\omega \cdot \theta}$

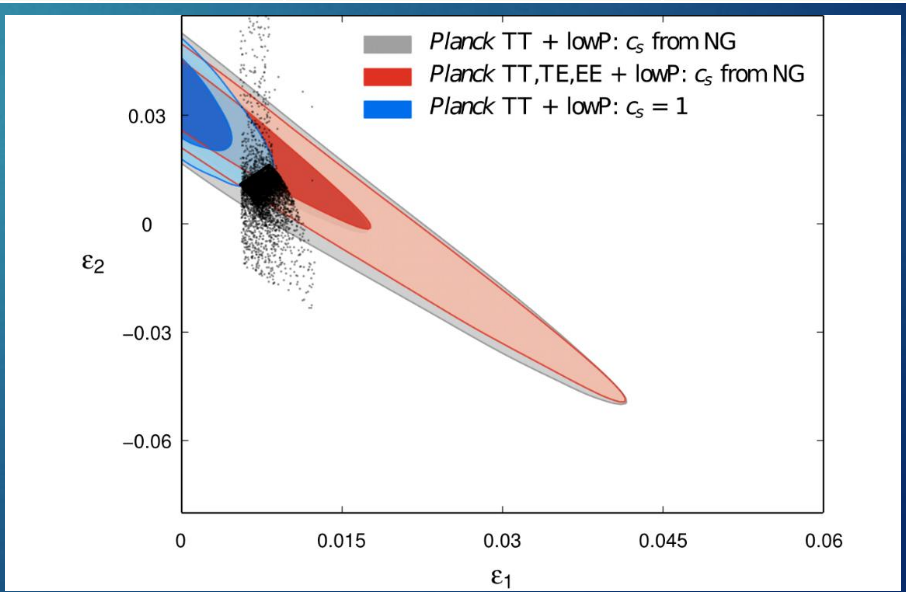
$60 < N < 90,\ 0 < \omega < 1$

Holographic tachyon cosmology

cosh	Mean	SD	Median	Min	Max
1st order	8.686E-04	5.779E-04	7.407E-04	4.345E-05	2.886E-03
Case 1	3.801E-03	2.495E-03	3.368E-03	1.563E-04	1.080E-02
Case 2	**3.075E-04**	1.511E-04	3.006E-04	3.950E-06	1.566E-03
Case 3a 3rd order	6.147E-02	2.318E-02	6.077E-02	1.713E-02	1.769E+00
Case 3a 2nd order	6.394E-02	1.197E-01	6.095E-02	2.524E-02	9.021E+00
Case 3b	2.584E-02	1.095E-02	2.404E-02	5.690E-03	5.555E-02

exp	Mean	SD	Median	Min	Max
1st order	2.109E-03	2.645E-04	2.078E-03	1.534E-03	3.100E-03
Case 1	9.503E-03	1.186E-03	9.349E-03	7.502E-03	1.331E-02
Case 2	**3.962E-04**	1.621E-04	3.735E-04	2.350E-06	1.744E-03
Case 3a 3rd order	6.554E-02	5.147E-03	6.502E-02	4.804E-02	1.563E-01
Case 3a 2nd order	6.678E-02	1.209E-02	6.533E-02	5.069E-02	5.208E-01
Case 3b	8.280E-03	2.082E-03	8.188E-03	6.156E-04	2.213E-02





Root mean square error (RMSE)

- Find "the best" model $\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{m}\left[\left(\frac{n_{si}-\bar{n}_s}{\bar{n}_s}\right)^2+\left(\frac{r_i-\bar{r}}{\bar{r}}\right)^2\right]}{m}}$
  - where m is number of simulations (n_{si}, r_i), $m \approx 10000$, and $(\bar{n}_s, \bar{r})$ are values determinated by the Planck collaboration $(\bar{n}_s = 0.9668, \bar{r} \approx 0.035)$.

Model		Potential	Analytic	1st order	Case 1	Case 2	Case 3a 2nd order	Case3b 3rd order
Tachyon	FLRW	exp	9.9075E-01	9.9075E-01	9.9081E-01	9.9075E-01	9.9066E-01	9.9066E-01
		cosh	9.8232E-01	9.8232E-01	9.8239E-01	9.8232E-01	9.8186E-01	9.8186E-01
	RSII	exp	9.9174E-01	9.9174E-01	9.9181E-01	9.9174E-01	9.9166E-01	9.9166E-01
		cosh	9.8866E-01	9.8866E-01	9.8873E-01	9.8866E-01	9.8846E-01	9.8846E-01
RSII model		exp	9.9149E-01	9.9149E-01	9.9155E-01	9.9149E-01	9.9139E-01	9.9141E-01
		cosh	9.8197E-01	9.8197E-01	9.8203E-01	9.8197E-01	9.8160E-01	9.8155E-01
Holography		exp	9.9139E-01	9.9141E-01	9.9147E-01	9.9139E-01	9.9081E-01	9.9107E-01
		cosh	9.5746E-01	9.5747E-01	9.5753E-01	9.5745E-01	9.5490E-01	9.5522E-01

Conclusion

- We discussed models of tachyon inflation based on a tachyonic, RSII and holographic braneworld scenario.
- We simulated observational parameters of inflation for two potentials
- The agreement of our model with the Planck observational data is good, especially for holographic model and a higher number of e-folds.
- Preliminary results showed that observational parameters (n_s, r) can be estimated very fast using the approximation.
- Preliminary results are promising and open good opportunity for further analytical research of these potentials and looking for a better approximation suitable to more different types of models.

Presentation available at: www.mmilan.com
Contact: milan.milosevic@pmf.edu.rs

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