

Keplerovi zakoni

(praktična primena)

od Marsa do supermasivne crne rupe

II deo

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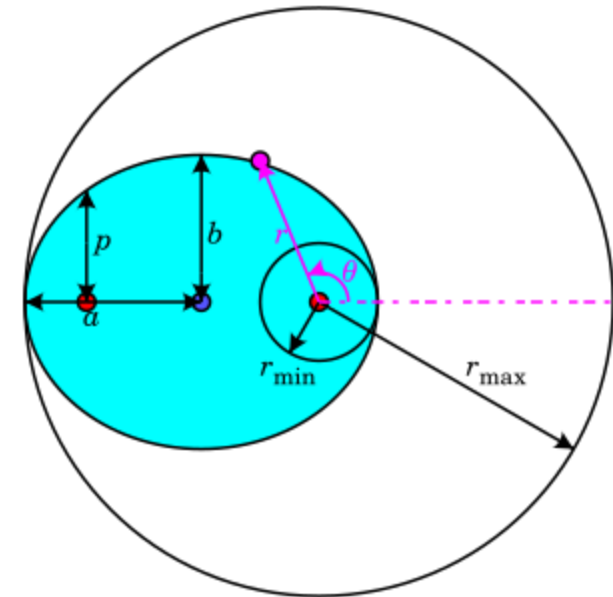
www.mmilan.com

Naučni klub Niš, 18. novembar 2025. god

1. Keplerov zakon

- ▶ **Orbita svake planete je elipsa, pri čemu se Sunce nalazi u jednoj od dve žiže.**
- ▶ Jednačina elipse $r = \frac{p}{1 + \varepsilon \cos \theta}$; ekscentricitet $\varepsilon = \sqrt{1 - \frac{b^2}{a^2}}$
 - ▶ gde je p orbitalni parametar, ε je ekscentricitet elipse, r je rastojanje od planete do Sunca, i θ je ugao trenutnog položaja planete meren u odnosu na pravac najbližeg položaja planete
 - ▶ Za $\theta = 90^\circ$ i $\theta = 270^\circ$, važi $r = p$.
- ▶ Aritmetička, geometrijska i harmonijska sredina:

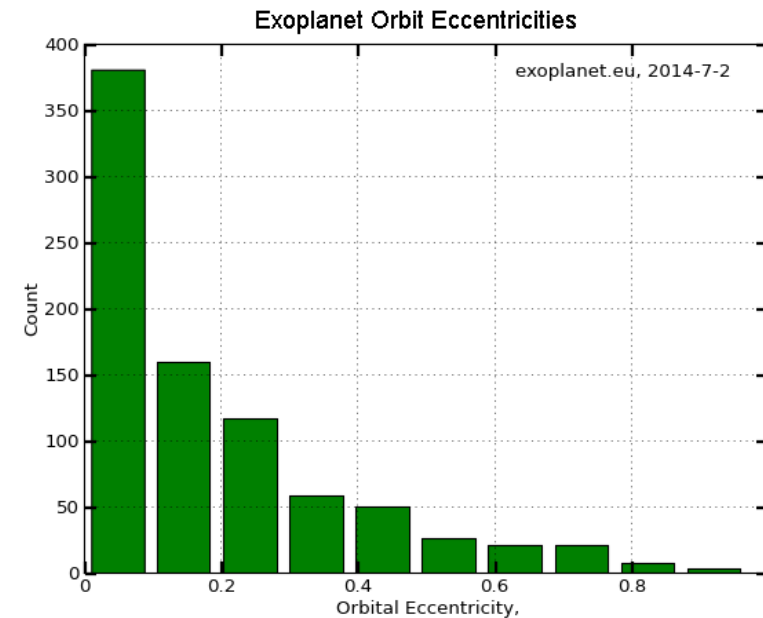
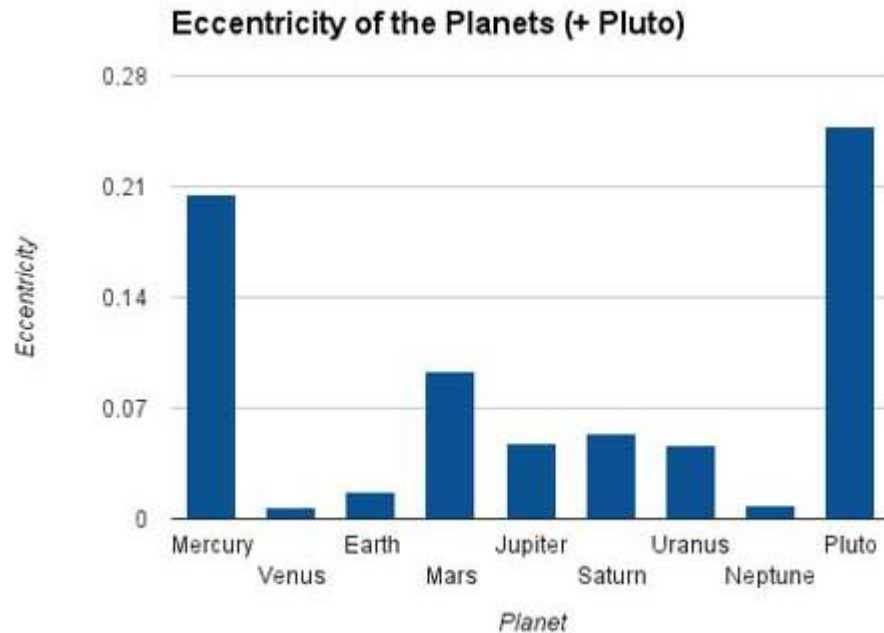
$$a = \frac{r_{\max} + r_{\min}}{2} = \frac{p}{1 - \varepsilon^2}$$
$$b = \sqrt{r_{\max} \cdot r_{\min}} = \frac{p}{\sqrt{1 - \varepsilon^2}}$$
$$p = \left(\frac{r_{\min}^{-1} + r_{\max}^{-1}}{2} \right)^{-1}$$



Heliocentrični (polarni) koordinatni sistem (r, θ) za elipsu. Na slici: velika polu-osa a , mala polu-osa b i orbitalni parametar (semi-latus rectum) p ; centar elipse i dve žiže obeležene su velikim tačkama. Za $\theta = 0^\circ$, $r = r_{\min}$ i za $\theta = 180^\circ$, $r = r_{\max}$.

1. Keplerov zakon

- ▶ Putanje planeta imaju mali ekscentricitet (nisu mnogo različite od kružnih), osim u slučaju Merkura i „bivše“ planete Pluton.
- ▶ Putanje prirodnih satelita, asteroida i periodičnih kometa takođe su eliptične.

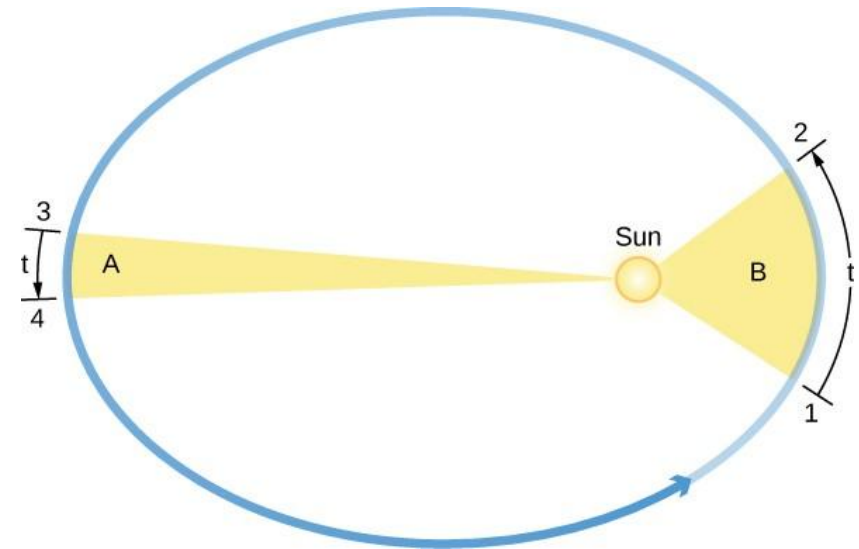
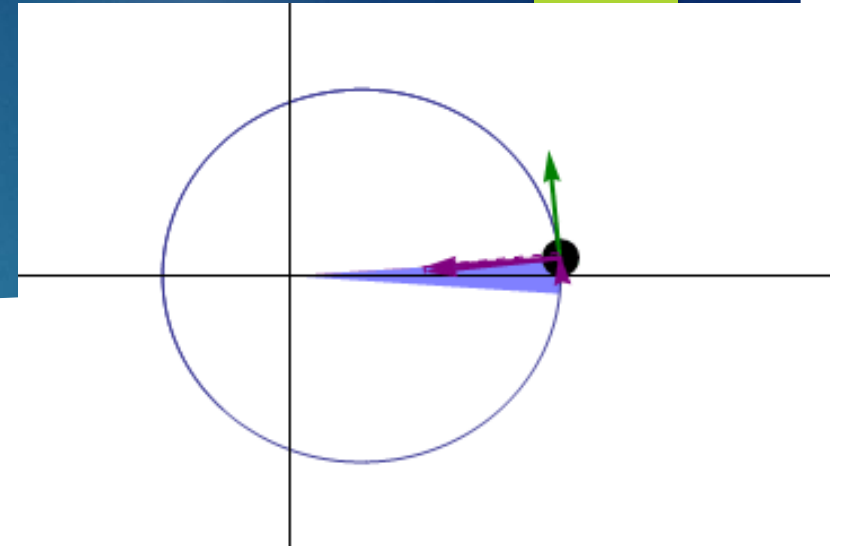


2. Keplerov zakon

- Prava koja spaja planetu i Sunce pokriva jednake površine u jednakim vremenskim intervalima.

$$\begin{aligned}dA &= \frac{1}{2} r \cdot r d\theta \\ \frac{dA}{dt} &= \frac{r^2}{2} \frac{d\theta}{dt} && \text{Konstantna arealna (sektorska) brzina} \\ T \cdot \frac{r^2}{2} \frac{d\theta}{dt} &= \pi ab \\ n = \frac{2\pi}{T} = \frac{360^\circ}{T} &\Rightarrow r^2 d\theta = ab \cdot n dt\end{aligned}$$

$$\boxed{\frac{dA}{dt} = \frac{abn}{2} = \frac{\pi ab}{n}}$$



3. Keplerov zakon

- Kvadrati perioda (T) obilaska planete oko Sunca srazmerni su kubovima velikih poluosa (a) njihovih putanja

$$\frac{a_1^3}{T_1^2} = \frac{a_2^3}{T_2^2} = \frac{a_3^3}{T_3^2} = \dots = \text{const}$$

- Koristeći **Njutnov zakon gravitacije** (1687), treći Keplerov zakon može se izvesti za slučaj kružne orbite tako što se centripetalna sila izjednači sa gravitacionom silom:

$$\begin{aligned} mr\omega^2 &= G \frac{mM}{r^2} \\ mr \left(\frac{2\pi}{T} \right)^2 &= G \frac{mM}{r^2} \\ T^2 &= \left(\frac{4\pi^2}{GM} \right) r^3 = \text{const} \cdot r^3 \end{aligned}$$

$$\frac{1}{(365.25)^2} \approx 7.5 \cdot 10^{-6}$$

$$\frac{a^3}{T^2} = \frac{G(M+m)}{4\pi^2} \approx \frac{GM}{4\pi^2} \approx 7.5 \cdot 10^{-6} \frac{\text{AU}^3}{\text{day}^2}$$

3. Keplerov zakon

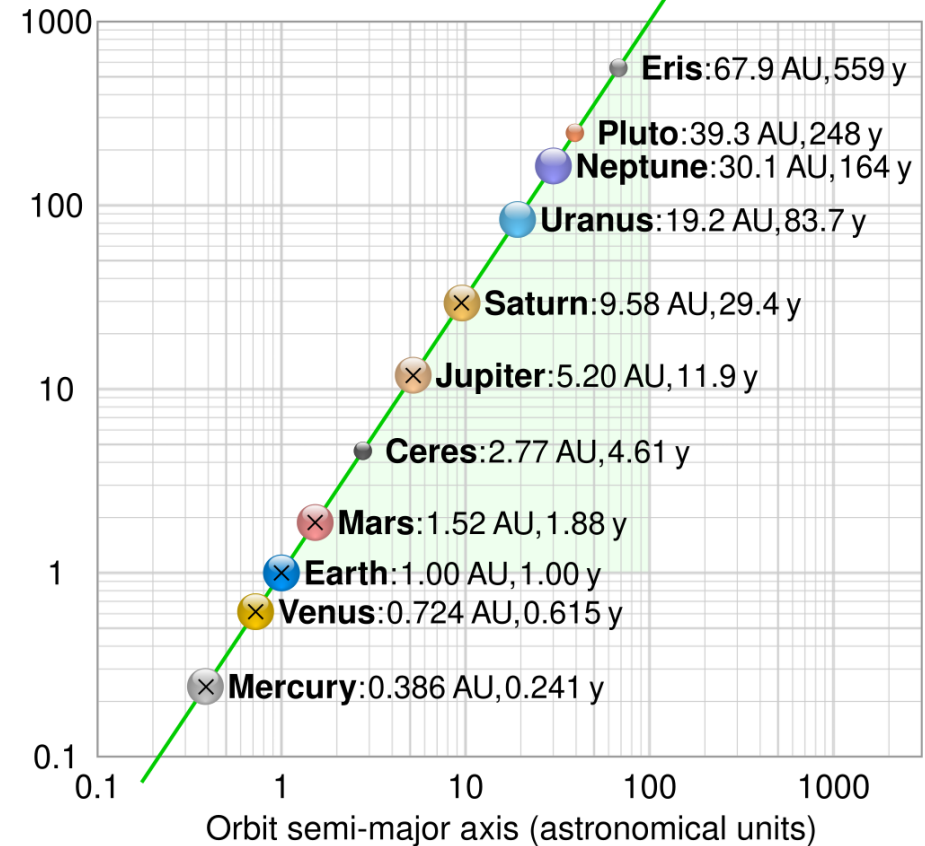
Data used by Kepler (1618)

Planet	Mean distance to sun (AU)	Period (days)	$\frac{R^3}{T^2}$ (10^{-6} AU ³ /day ²)
Mercury	0.389	87.77	7.64
Venus	0.724	224.70	7.52
Earth	1	365.25	7.50
Mars	1.524	686.95	7.50
Jupiter	5.2	4332.62	7.49
Saturn	9.510	10759.2	7.43

Modern data (Wolfram Alpha Knowledgebase 2018)

Planet	Semi-major axis (AU)	Period (days)	$\frac{R^3}{T^2}$ (10^{-6} AU ³ /day ²)
Mercury	0.38710	87.9693	7.496
Venus	0.72333	224.7008	7.496
Earth	1	365.2564	7.496
Mars	1.52366	686.9796	7.495
Jupiter	5.20336	4332.8201	7.504
Saturn	9.53707	10775.599	7.498
Uranus	19.1913	30687.153	7.506
Neptune	30.0690	60190.03	7.504

Orbital period (Earth years)





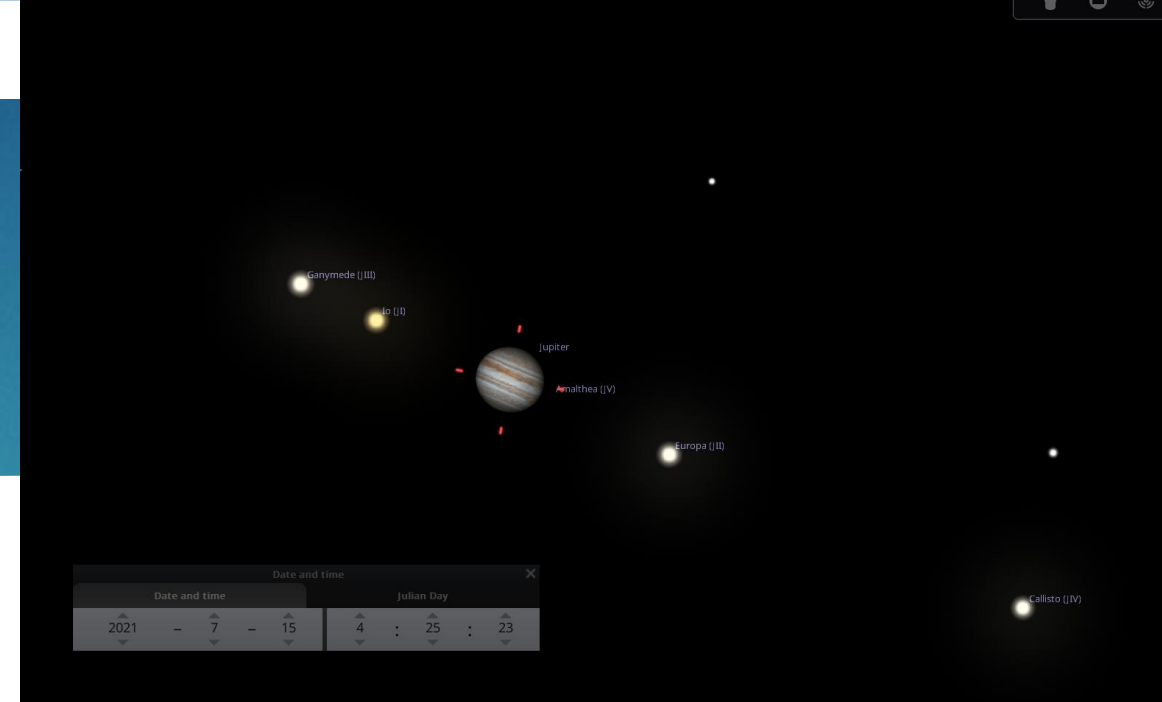
Masa Jupitera

Masa Jupitera

- ▶ Ne možemo direktno izmeriti sve u svemiru - mase i udaljenosti planeta i njihovih meseca.
- ▶ Međutim, iz njihovih kretanja možemo izvesti i zaključiti mnoge osobine nebeskih tela, iako ih ne možemo neposredno izmeriti.
- ▶ Treći Keplerov zakon: $\frac{T^2}{a^3} = \frac{4\pi^2}{G \cdot M}$

$$M = \frac{a^3}{T^2}$$

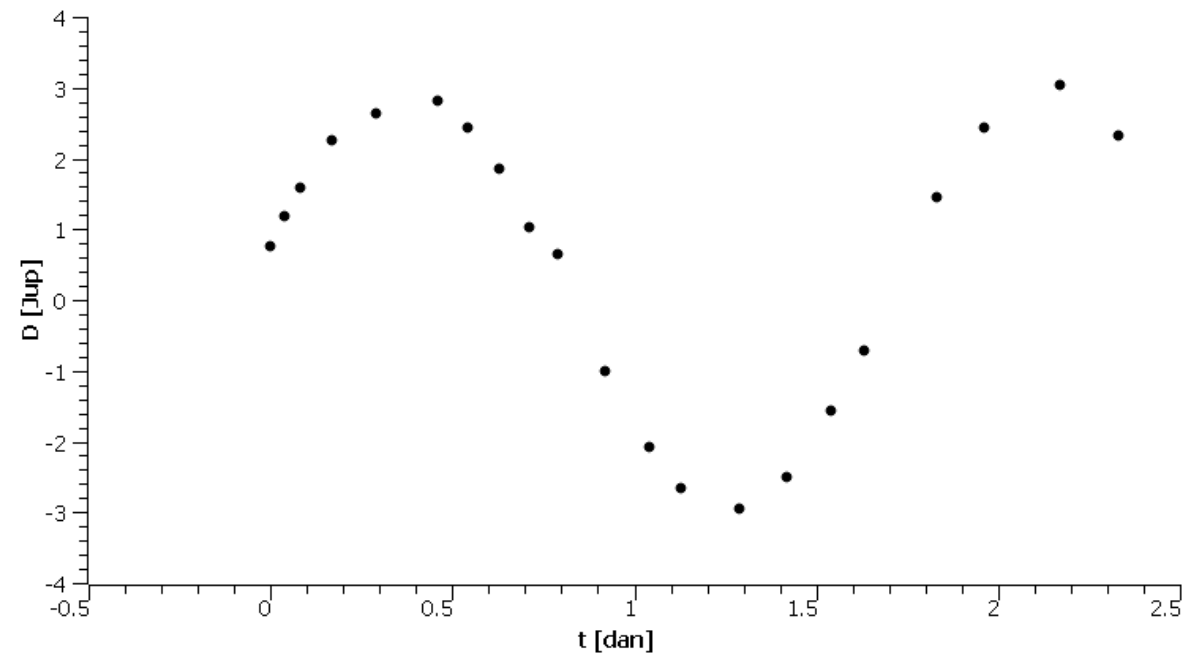
- ▶ M – masa centralnog tela (u $M_{\odot}$),
- ▶ a – velika polu-osa (u AU),
- ▶ T – period (u godinama)



Primer (Io)

„Posmatranje“ (Stellarium)

time	ΔT	sig	min	sec	Jupiter	D [Jup]
2459409.565	0.00	1	0	35.41	46.51	0.7613
2459409.607	0.04	1	0	55.26	46.51	1.1881
2459409.649	0.08	1	1	13.87	46.51	1.5883
2459409.732	0.17	1	1	44.86	46.51	2.2546
2459409.857	0.29	1	2	2.12	46.51	2.6257
2459410.024	0.46	1	2	11.16	46.51	2.8200
2459410.107	0.54	1	1	53.27	46.51	2.4354
2459410.191	0.63	1	1	26.67	46.51	1.8635
2459410.274	0.71	1	0	47.87	46.51	1.0292
2459410.357	0.79	1	0	30.00	46.51	0.6450
2459410.482	0.92	-1	0	47.08	46.51	-1.0123
2459410.607	1.04	-1	1	36.94	46.51	-2.0843
2459410.691	1.13	-1	2	3.28	46.51	-2.6506
2459410.857	1.29	-1	2	16.76	46.51	-2.9404
2459410.982	1.42	-1	1	56.61	46.51	-2.5072
2459411.107	1.54	-1	1	12.66	46.51	-1.5622
2459411.191	1.63	-1	0	33.35	46.51	-0.7171
2459411.399	1.83	1	1	7.69	46.51	1.4554
2459411.524	1.96	1	1	53.30	46.51	2.4360
2459411.732	2.17	1	2	21.25	46.51	3.0370
2459411.899	2.33	1	1	48.32	46.51	2.3290



Posmatranje (rezultati)

$$y = a \cos(\omega (t - c)) + b$$

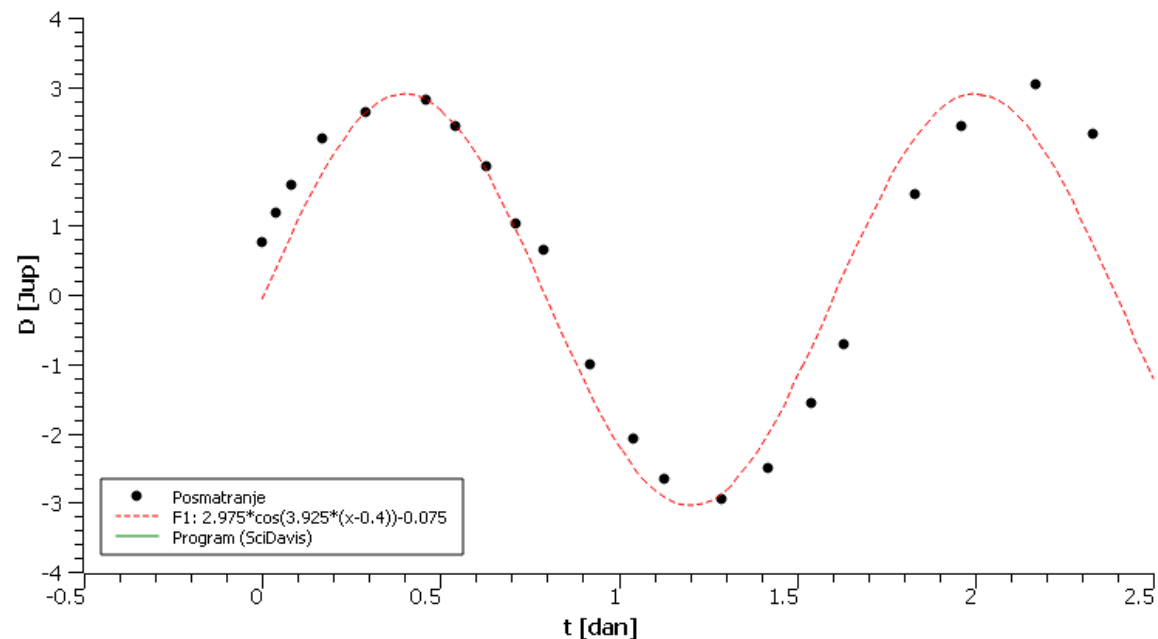
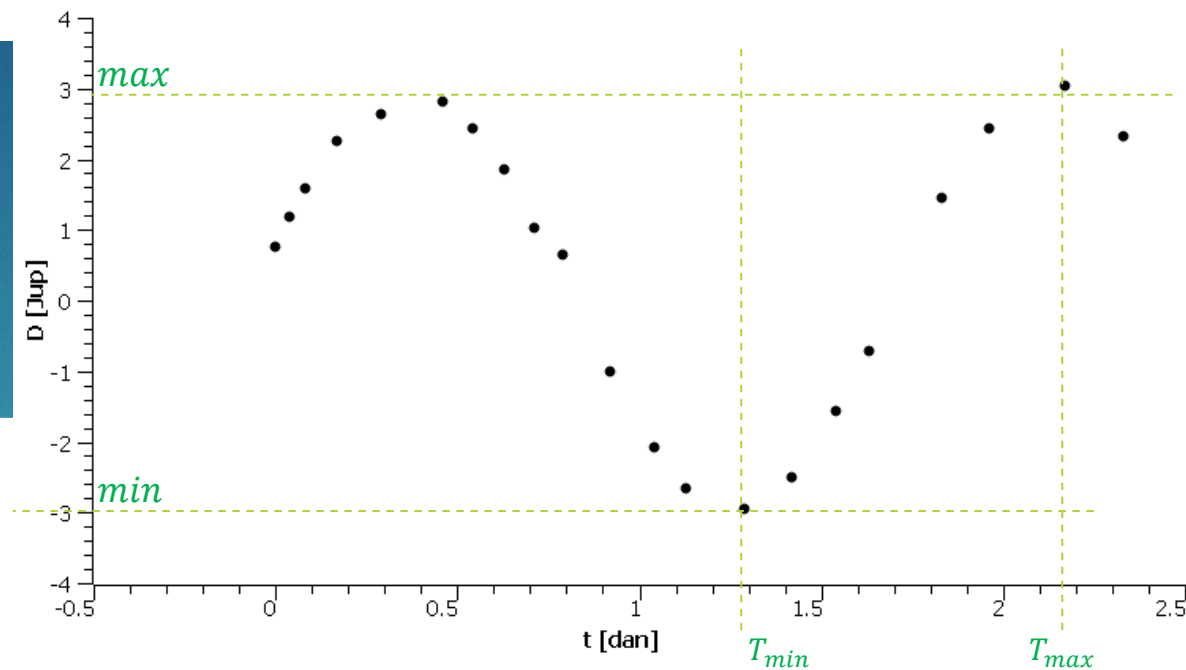
$$b = \frac{\max + \min}{2} = \frac{1}{2}(2.9 + (-3.05)) = \frac{-0.15}{2} = -0.075$$

$$a = \frac{\max - \min}{2} = \frac{1}{2}(2.9 - (-3.05)) = \frac{5.95}{2} = 2.975$$

$$T = \frac{2\pi}{\omega} = 2(T_{\max} - T_{\min}) = 2(2.1 - 1.3) = 2 \cdot 0.8 = 1.6$$

$$\omega = \frac{2\pi}{T} = 3.925$$

$$c = 0.4$$



Bolji način (SciDavis)

Function: user1 (x, a, b, c, w)

$a * \cos(w * (x - c)) + b$

Parameter	Value	Constant
a	-2.955786850733	☐
b	0.0408699848125348	☐
c	1.25869498609216	☐
w	-3.55636264712317	☐

Initial guesses

Algorithm: Scaled Levenberg-Marquardt

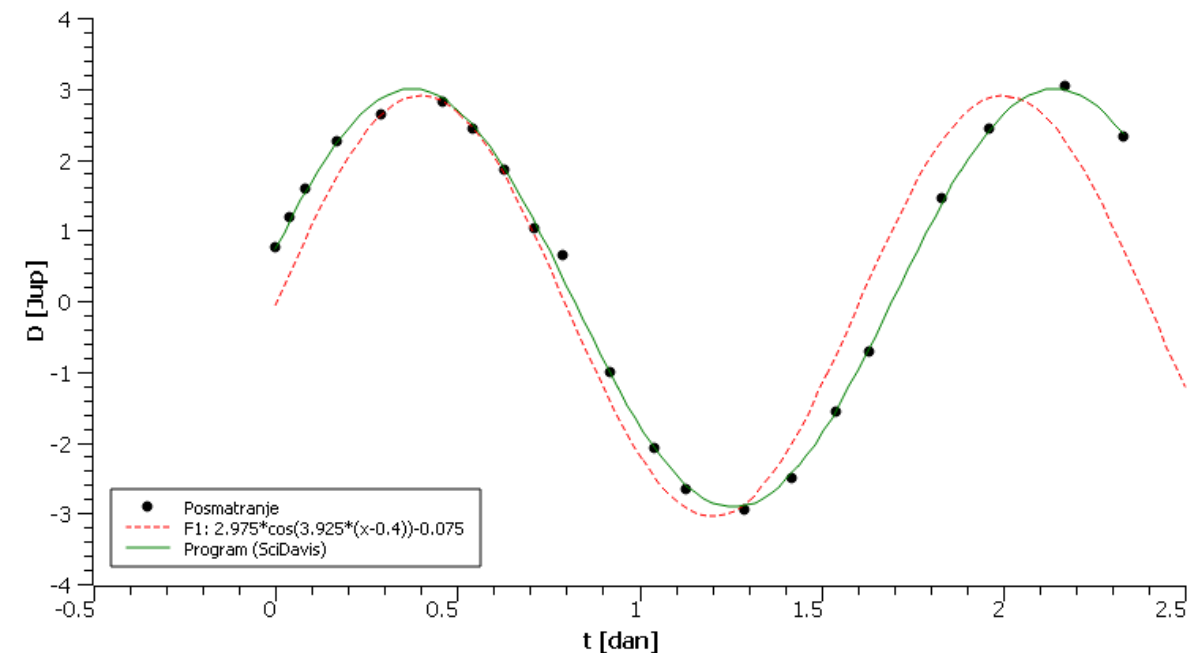
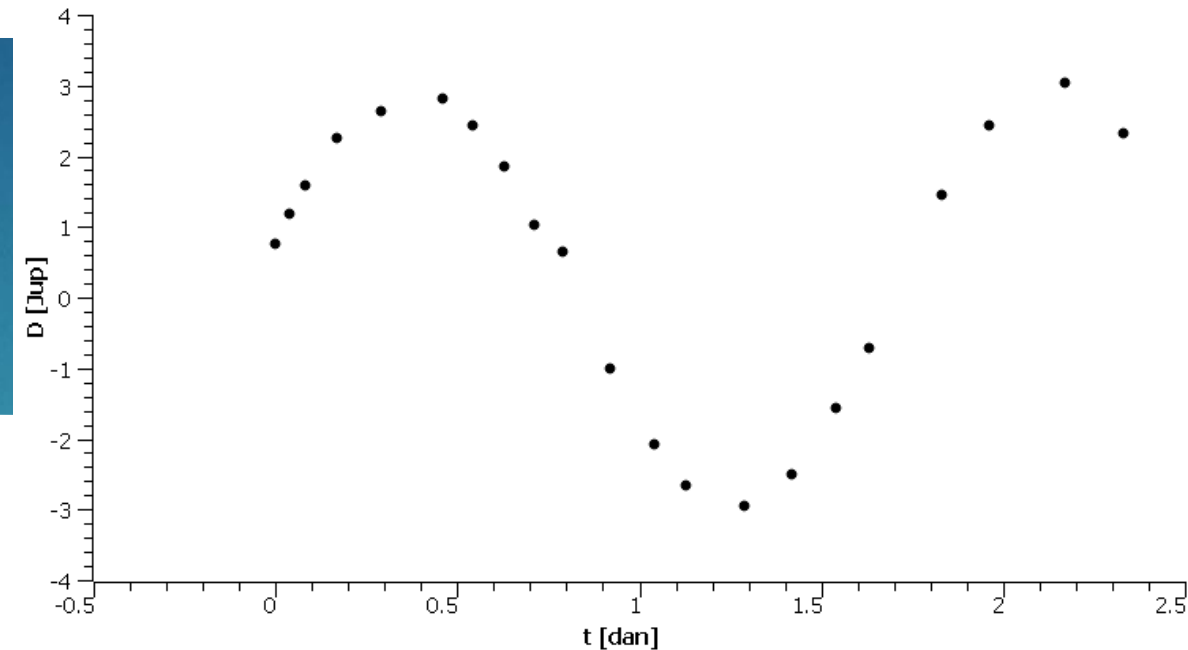
Color:  

From x= 0 To x= 2.33

Iterations: 1000 Tolerance: 1e-4

<https://scidavis.sourceforge.net/>

$$b = -0.075, a = 2.975, \omega = 3.925, c = 0.4$$



Hajde da uporedimo...

► „Papir i olovka“

► $a = 2.956 \text{ [Jup]} = 0.00276 \text{ AU}$

► $T = 1.6 \text{ dana} = 0.0044 \text{ godina}$ $\Delta = 4.29 \%$

► $M = 0.0009134 M_{\odot}$

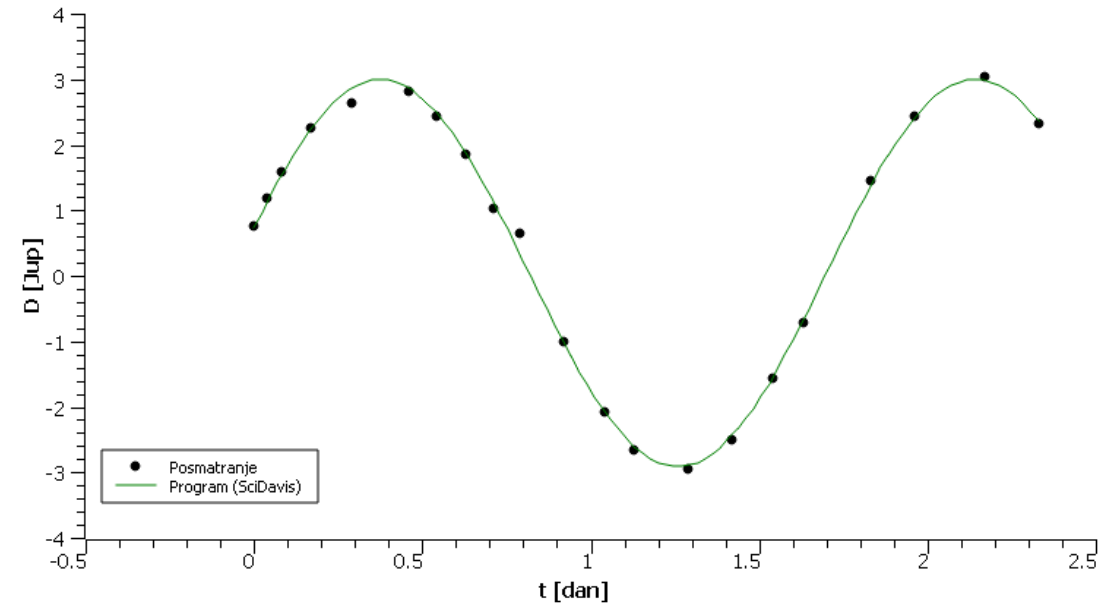
$$M = \frac{a^3}{T^2}$$

► „Naučni“ metod

► $a = 2.975 \text{ [Jup]} = 0.00278 \text{ AU}$

► $T = 1.766 \text{ day} = 0.0048 \text{ year}$ $\Delta = 2.43 \%$

► $M = 0.0009311 M_{\odot}$

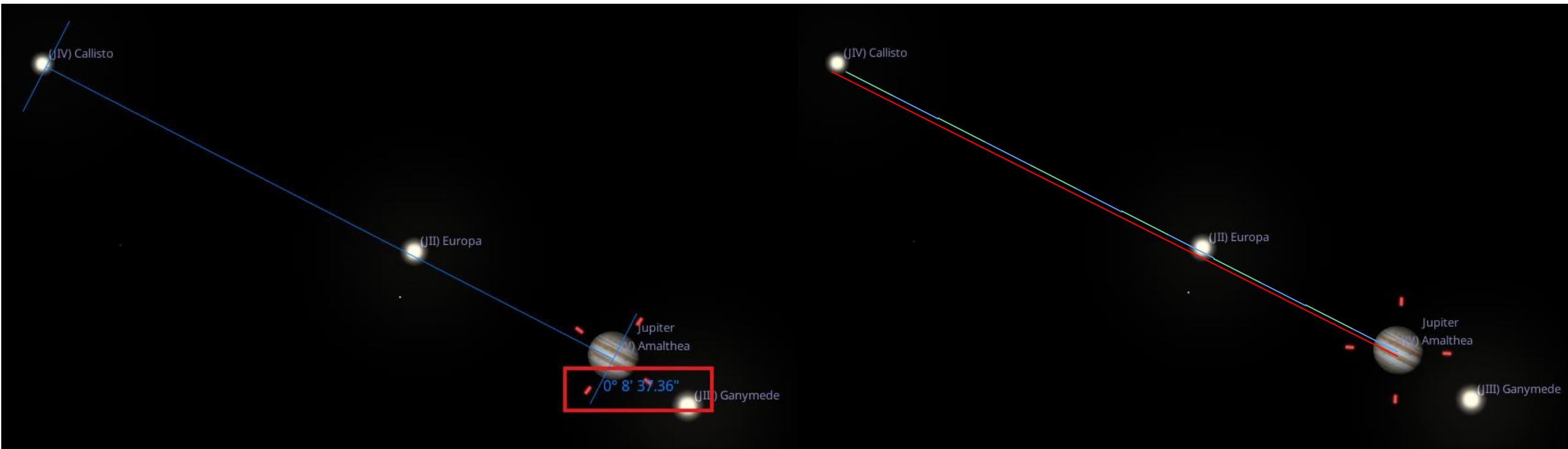


$$M = 0.0009543 M_{\odot}$$



Simuliranje posmatranja i obrada rezultata

Kako meriti?



Korisne formule

Masa Jupitera - lo ☆ 📁 ☁

File Edit View Insert Format Data Tools Extensions Help

Search Menus 100% 123 Default... 10 B

G2 $=C2*(D2*60+E2)/F2$

	A	B	C	D	E	F	G	H	I	J	K	L
	vreme	T	sig	Min	sec	Jupiter	D [Jup]	D [AU]				
2	2459409.565	0	1	0	35.41	46.51	0.761341647	7.12E-04		Jupiter [km]	139820	
3	2459409.607	0.04	1	0	55.26	46.51	1.188131585	1.11E-03		AU [km]	149597871	
4	2459409.649	0.08	1	1	13.87	46.51	1.588260589	1.48E-03		Jupiter [AU]	9.35E-04	
5	2459409.732	0.17	1	1	44.86	46.51	2.25456891	2.11E-03				
6	2459409.857	0.29	1	2	2.12	46.51	2.625671899	2.45E-03				
7	2459410.024	0.46	1	2	11.16	46.51	2.820038701	2.64E-03				
8	2459410.107	0.54	1	1	53.27	46.51	2.435390239	2.28E-03				
9	2459410.191	0.63	1	1	26.67	46.51	1.863470221	1.74E-03				
10	2459410.274	0.71	1	0	47.87	46.51	1.029241023	9.62E-04				
11	2459410.357	0.79	1	0	30	46.51	0.6450225758	6.03E-04				
12	2459410.482	0.92	-1	0	47.08	46.51	-1.012255429	-9.46E-04				
13	2459410.607	1.04	-1	1	36.94	46.51	-2.08428295	-1.95E-03				
14	2459410.691	1.13	-1	2	3.28	46.51	-2.650612771	-2.48E-03				
15	2459410.857	1.29	-1	2	16.76	46.51	-2.940442916	-2.75E-03				
16	2459410.982	1.42	-1	1	56.61	46.51	-2.507202752	-2.34E-03				
17	2459411.107	1.54	-1	1	12.66	46.51	-1.562244679	-1.46E-03				
18	2459411.191	1.63	-1	0	33.35	46.51	-0.7170500968	-6.70E-04				
19	2459411.399	1.83	1	1	7.69	46.51	1.455385939	1.36E-03				
20	2459411.524	1.96	1	1	53.3	46.51	2.436035261	2.28E-03				
21	2459411.732	2.17	1	2	21.25	46.51	3.036981294	2.84E-03				
22	2459411.899	2.33	1	1	48.32	46.51	2.328961514	2.18E-03				

Convert to table

Ctrl + D

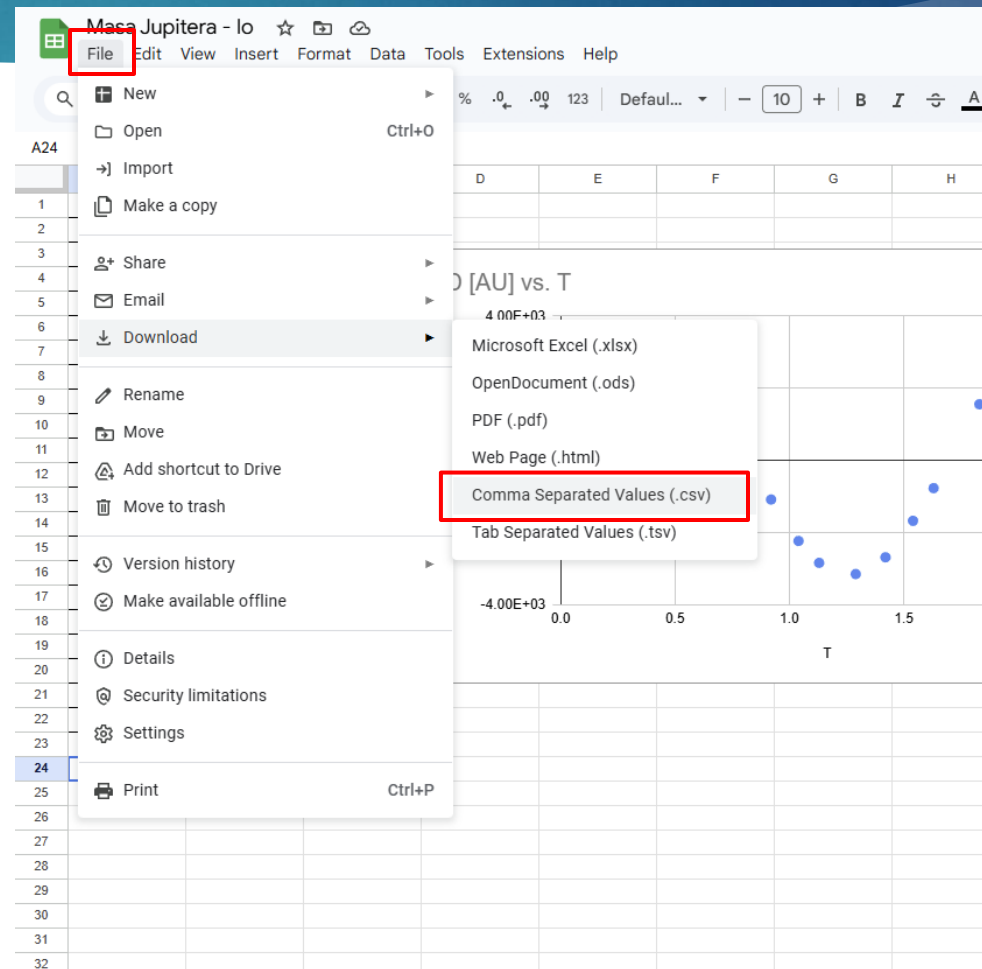
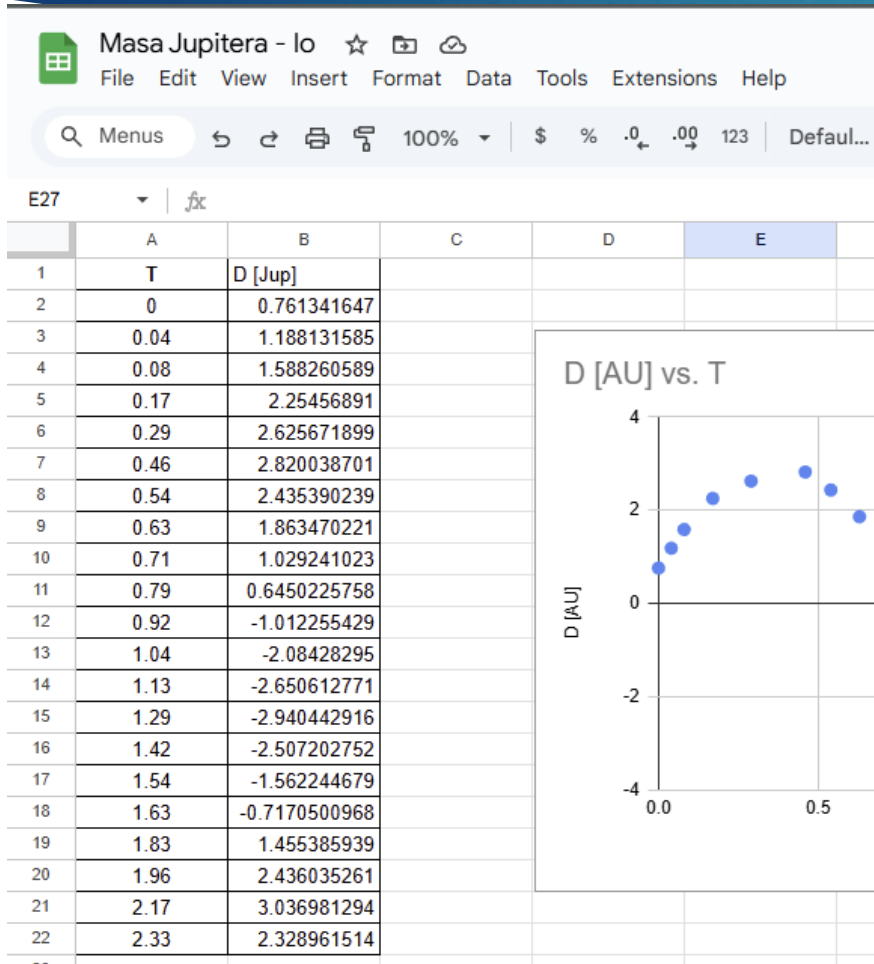
$$r_{Jup} [km] = 139\,820 \text{ km}$$

$$r_{Jup} [AU] = \frac{r_{Jup} [km]}{1 \text{ AU}_{[km]}}$$

$$D_{[Jup]} = znak \cdot \frac{60 \cdot arcmin + arcsec}{r_{[arcsec]}}$$

Tabela sa podacima

<https://docs.google.com/>



Crtanje grafika

<https://www.geogebra.org/calculator>

GeoGebra Calculator Suite

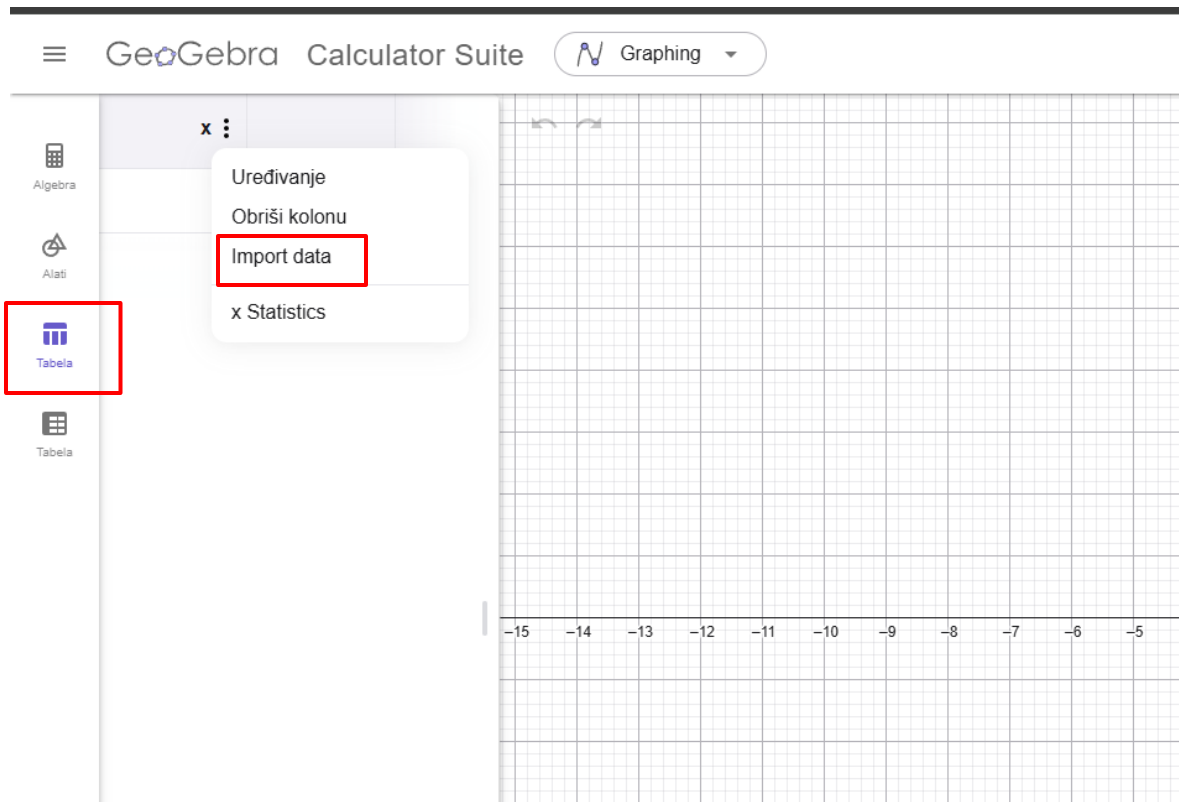
Algebra

Alati

Tabela

x :

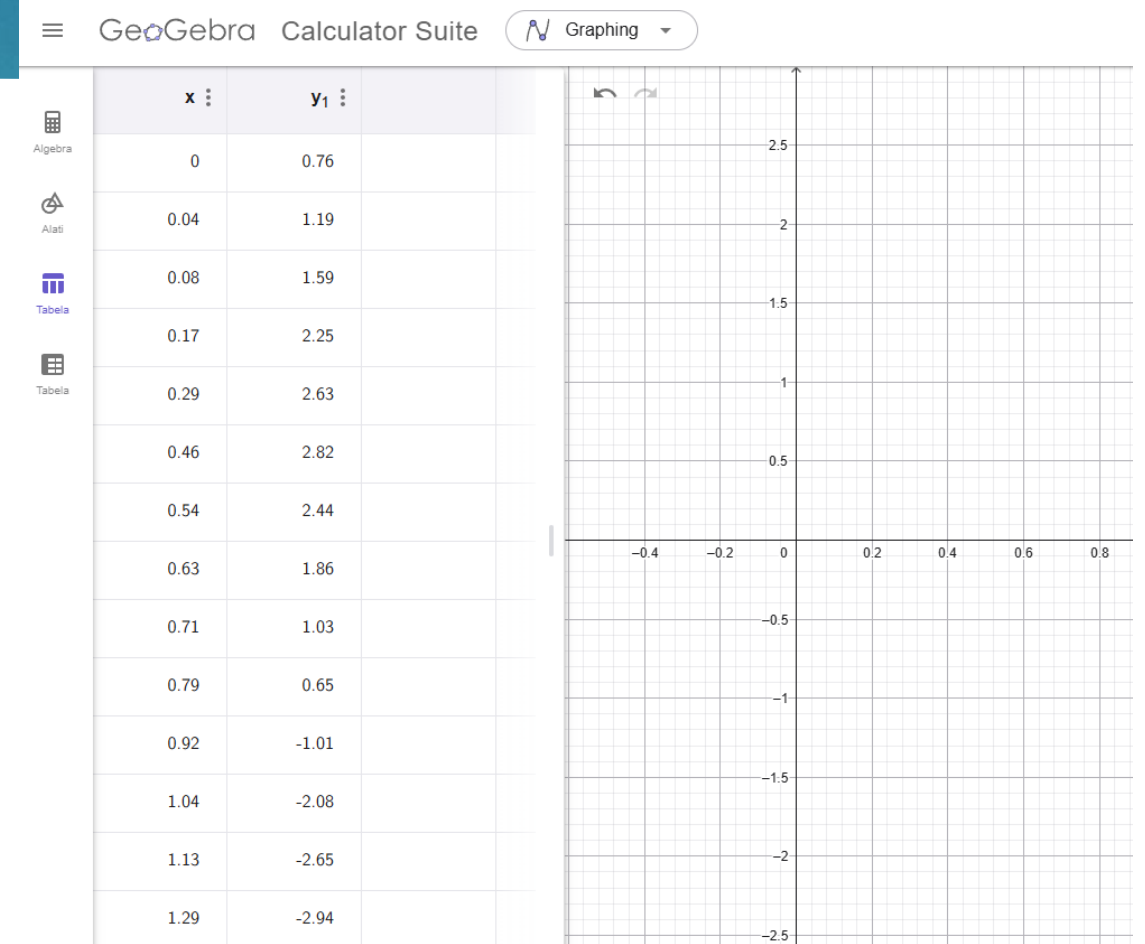
- Uređivanje
- Obriši kolonu
- Import data
- x Statistics



GeoGebra Calculator Suite

Graphing

x :	y ₁ :
0	0.76
0.04	1.19
0.08	1.59
0.17	2.25
0.29	2.63
0.46	2.82
0.54	2.44
0.63	1.86
0.71	1.03
0.79	0.65
0.92	-1.01
1.04	-2.08
1.13	-2.65
1.29	-2.94



Crtanje grafika

<https://www.geogebra.org/calculator>

GeoGebra Calculator Suite Graphing

ASSIGN PRIJAVITE SE

Tabela

x	y ₁
0	0.7
0.04	1.19
0.08	1.59
0.17	2.25
0.29	2.63
0.46	2.82
0.54	2.44
0.63	1.86
0.71	1.03
0.79	0.65
0.92	-1.01
1.04	-2.08
1.13	-2.65
1.29	-2.94
1.42	-2.51

Graphing

Dimenzije

x Min: -0.60974
x Max: 3.60384
y Min: -3
y Max: 3

xOsa : yOsa
1 : 2.39906

Koordinatne ose

☒ Prikaži ose ☐ Masno
Boja: Vrsta linije: →
Label Style: ☐ Serif
☐ Masno ☐ Koso

Traka za korake konstrukcije

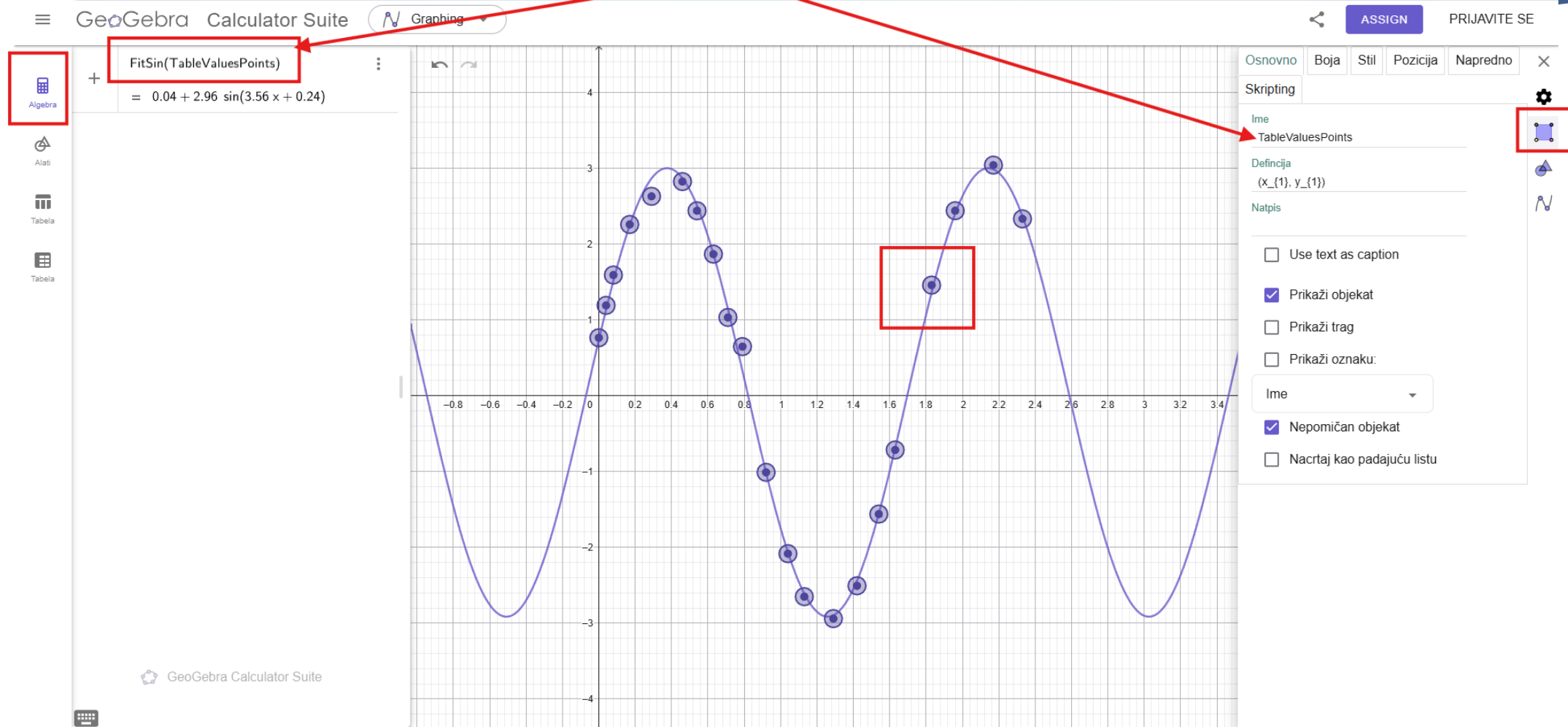
☐ Prikaži
☒ Dugme za pokretanje
☒ Dugme za opis konstrukcije

Razno

Boja pozadine:

Crtanje grafika

<https://www.geogebra.org/calculator>



Crtanje grafika

<https://www.geogebra.org/calculator>

GeoGebra Calculator Suite

Graphing



ASSIGN

PRIJAVITE SE

Algebra

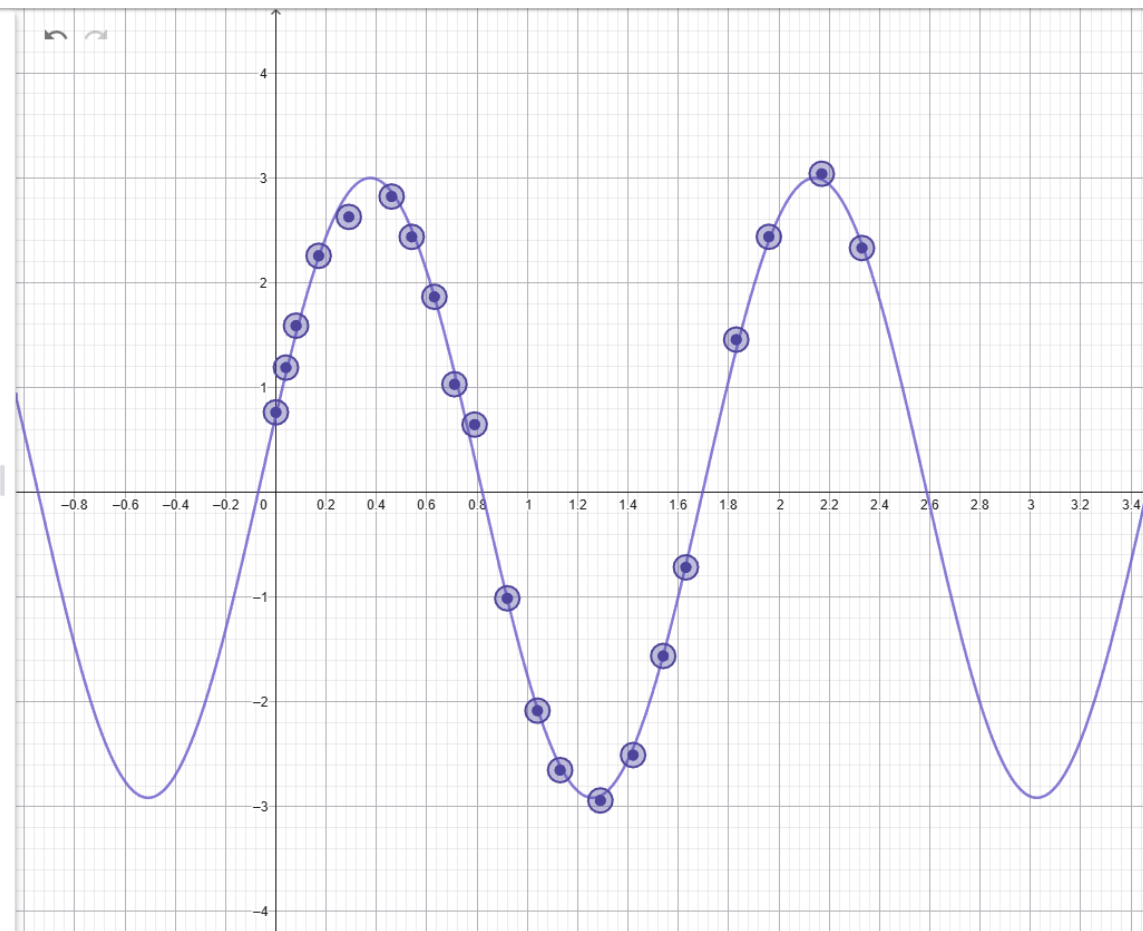
Alati

Tabela

Tabela

FitSin(TableValuesPoints)

$$= 0.04 + 2.96 \sin(3.56 x + 0.24)$$



GeoGebra Calculator Suite

Global

Jezik: Serbian / српски

Zaokruživanje:

2 decimal. mesta

Označavanje:

Za sve nove objekte

Veličina slova:

16 pt

SNIMI PODEŠAVANJE

RESTORE SETTINGS

Određivanje mase Jupitera

- ▶ $f(x) = 0.04 + 2.96 \cdot \sin(3.56 x + 0.24)$
- ▶ $f(x) = 0.04 + 2.96 \cdot \sin\left(3.56 \left(x + \frac{0.24}{3.56}\right)\right)$
- ▶ Amplituda $a_{[Jup]} = 2.96$
- ▶ Period $T_{[dan]} = \frac{2\pi}{3.56} = 1.76$
- ▶ Masa Jupitera $M_J = \frac{(2.96 \cdot 9.35 \cdot 10^{-4})^3}{\left(1.76 \cdot \frac{1}{365.25}\right)^2} = \frac{(0.0028)^3}{(0.0048)^2} = \frac{2.12 \cdot 10^{-8}}{2.30 \cdot 10^{-5}}$

$$M_J = 0.0009201 M_{\odot}$$



Masa SMBH

 OTHERS ▾
 [HELP ?](#)

Catalog







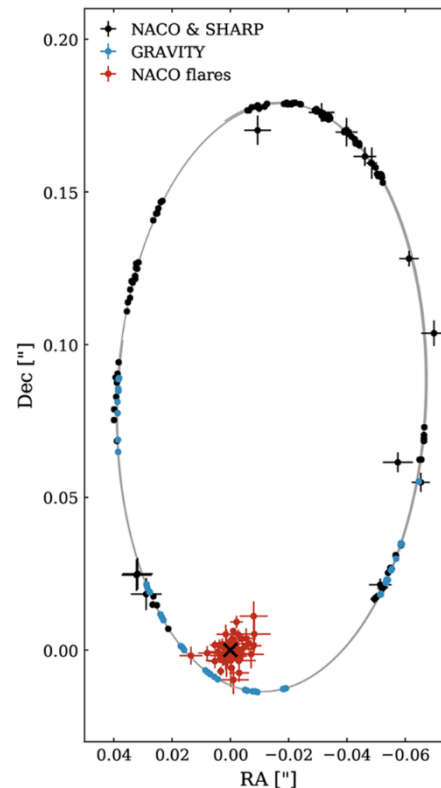

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[J/ApJ/707/L114/table1](#) [The orbit of S2 star around Sgr A* \(Gillessen+, 2009\)](#) [ReadMe!ftp](#)
[Post annotation](#) Orbital data for S2 used in this work ([C/](#), [velocity](#) [timeSerie](#) [2009ApJ...707L.114G](#)
 and [RA/](#) [astrometry](#)) (96 rows)

Vežba – izračunajte masu SMBH

- ▶ Desno – koordinate položaja zvezde S2
- ▶ Koordinatni početak – centar SMBH
- ▶ Kako?
 - ▶ Nacrtati tačke (i greške)
 - ▶ Nacrtati elipsu najpribližniju merenjima
 - ▶ Izmeriti veliku poluosu elipse
 - ▶ Arcsec prevesti u svetlosne dane (ld),
 $2 \text{ arcsec} = 28 ld$
 - ▶ Izračunati srednju vrednost ☺



Date (year)	x (arcsec)	dx (arcsec)	y (arcsec)	dy (arcsec)
1992.226	0.104	0.003	-0.166	0.004
1994.321	0.097	0.003	-0.189	0.004
1995.531	0.087	0.002	-0.192	0.003
1996.256	0.075	0.007	-0.197	0.010
1996.428	0.077	0.002	-0.193	0.003
1997.543	0.052	0.004	-0.183	0.006
1998.365	0.036	0.001	-0.167	0.002
1999.465	0.022	0.004	-0.156	0.006
2000.474	-0.000	0.002	-0.103	0.003
2000.523	-0.013	0.003	-0.113	0.004
2001.502	-0.026	0.002	-0.068	0.003
2002.252	-0.013	0.005	0.003	0.007
2002.334	-0.007	0.003	0.016	0.004
2002.408	0.009	0.003	0.023	0.005
2002.575	0.032	0.002	0.016	0.003
2002.650	0.037	0.002	0.009	0.003
2003.214	0.072	0.001	-0.024	0.002
2003.353	0.077	0.002	-0.030	0.002
2003.454	0.081	0.002	-0.036	0.002

Vežba – izračunajte masu SMBH

► ...

► Odrediti period (P)

► $A_{ell} = ab \cdot \pi$ - a i b sa slike, prethodni slajd

► $\Delta A = \frac{\Delta t}{P} \cdot A_{ell}$ - II Keplerov zakon

► Nepoznato ΔA , Δt , A_{ell}

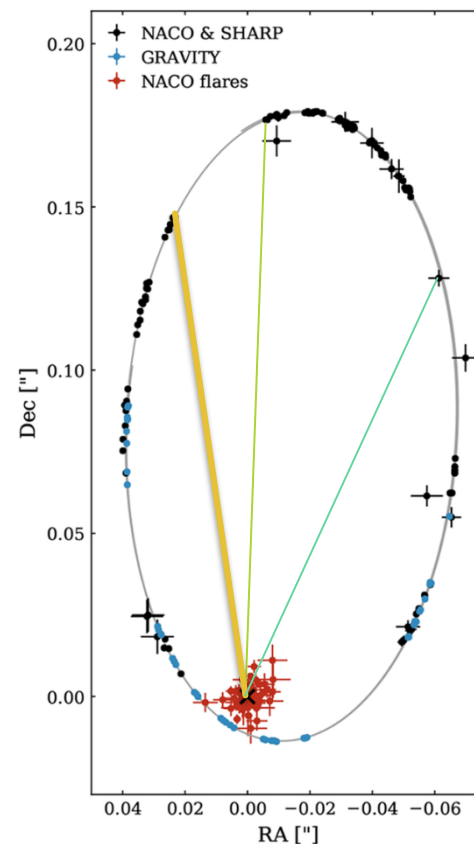
► ΔA i Δt – sa slike (prethodni slajd, za svaki segment)

► Nacrtati trougao i odrediti njegovu površinu (ponoviti više puta!)

► Izračunati masu SMBH

► III Keplerov zakon

► $P^2 = \frac{4\pi^2}{G(M+m_{S2})} a^3$, $M \gg m_{S2}$



Date (year)	x (arcsec)	dx (arcsec)	y (arcsec)	dy (arcsec)
1992.226	0.104	0.003	-0.166	0.004
1994.321	0.097	0.003	-0.189	0.004
1995.531	0.087	0.002	-0.192	0.003
1996.256	0.075	0.007	-0.197	0.010
1996.428	0.077	0.002	-0.193	0.003
1997.543	0.052	0.004	-0.183	0.006
1998.365	0.036	0.001	-0.167	0.002
1999.465	0.022	0.004	-0.156	0.006
2000.474	-0.000	0.002	-0.103	0.003
2000.523	-0.013	0.003	-0.113	0.004
2001.502	-0.026	0.002	-0.068	0.003
2002.252	-0.013	0.005	0.003	0.007
2002.334	-0.007	0.003	0.016	0.004
2002.408	0.009	0.003	0.023	0.005
2002.575	0.032	0.002	0.016	0.003
2002.650	0.037	0.002	0.009	0.003
2003.214	0.072	0.001	-0.024	0.002
2003.353	0.077	0.002	-0.030	0.002
2003.454	0.081	0.002	-0.036	0.002

Preuzimanje podataka

<https://vizier.cds.unistra.fr/viz-bin/VizieR>



VizieR

Search Criteria

Preferences

max: 50

HTML Table

☐ All columns

Compute

Mirrors

CDS, France

[VizieR home](#) · [Photometry viewer](#) · [Query VizieR using TAP](#) · [X-match tables](#) · [Query images/spectra](#)
The VizieR service is now hosted by CDS domain (cds.unistra.fr). Please, modify your configuration for the new domain.

Find catalogs among 26832 available

Clear Find...
Expand search ☐

? Catalog, author's name, word(s) from title, description, etc.
e.g.: AGN, Veron, I/239, or bibcodes...

► Search for catalogs by column descriptions (UCD) **?**

► Search for catalogs containing additional data

Search by Position across 30452 tables

Target Name (resolved by [Sesame](#)) or Position: Target dimension: Go!

NB: The epoch used for the query is the original epoch of the table(s) ☒ Radius ☐ Box size

i More about VizieR

sort by : popularity ☐ date ☐

Tools related to VizieR

- [Catalogue collection](#) : Search VizieR catalogues available via various services (FTP, VizieR, TAP, ...)
- [CDS Portal](#) : Access CDS data including VizieR, Simbad and Aladin using the CDS portal
- [Spectra, images in VizieR](#) : Search Spectra, images in VizieR
- [Photometry viewer](#) : Plot photometry (sed) including all VizieR
- [TAP VizieR](#) : query VizieR using ADQL (a SQL extension dedicated for astronomy)
- [CDS cross-match service](#) : fast cross-identification between any 2 tables, including VizieR catalogues, SIMBAD

→ [Cite/acknowledged](#)
→ [Rules of usage](#)

1

2



Catalog

J/ApJ/707/L114

The orbit of S2 star around Sgr A* (Gillessen+, 2009)

[First observation](#)

[2009ApJ...171L114G](#)

[ReadMe-ftp](#)

J/ApJ/707/L114 table1

Orbital data for S2 used in this work [timeSeries] (velocity and astrometry) (96 rows)

Simple Constraint

List Of Constraints

Query by Constraints applied on Columns (Output Order: + -)

Submit

Reset All

Show	Sort	Column	Constraint	Explain (UCD)
☐	☐	record		Record number assigned by the VizieR team. Should Not be used for identification. (meta:record)
☒	☐	Ep.A	yr	Year of the astrometric observation (time:epoch)
☒	☐	oRA	mas	Offset in Right Ascension relative to SgrA* (pos.eq.ra:arith.diff)
☒	☐	e_oRA	mas	Uncertainty in oRA (stat:error)
☒	☐	oDE	mas	Offset in Declination relative to SgrA* (pos.eq.dec:arith.diff)
☒	☐	e_oDE	mas	Uncertainty in oDE (stat:error)
☒	☐	Tel.A	(char)	Telescope used in astrometric observation (meta:id:instr.tel)
☒	☐	Ep.V	yr	(a) Year of the radial velocity observation (time:epoch)
☒	☐	VLSR	km/s	(a) Local Standard of Rest Velocity (physa:veloc:pos.lsr)
☒	☐	e_VLSR	km/s	(a) Uncertainty in VLSR (stat:error:physa:veloc:pos.lsr)
☒	☐	Tel.V	(char)	Telescope used in radial velocity observation (VLT or Keck) (meta:id:instr.tel)

☐ ALL col.

Adapt form

Display your selection only

Reset to default columns

☒ UCD1+ ☐ UCD1



→ [Cite/acknowledge VizieR catalogue](#)
→ [Rules of usage of VizieR data](#)

[Contact](#) - [Legals](#)

Preuzimanje podataka

<https://vizier.cds.unistra.fr/viz-bin/VizieR>



1

Search Criteria

Keywords: JApJ/707/L114

Tables: JApJ/707/L114/table1

Preferences: max: 50, HTML Table, All columns, Compute

Mirrors: CDS, France

Show constraint information

JApJ/707/L114/table1

The orbit of S2 star around Sgr A* (Gillessen+, 2009)

Orbital data for S2 used in this work (velocity and astrometry) (96 rows)

plot the output query using TAP:SQL

Full	Ep.A	oRA	e	oDE	e	Tel.A	Ep.V	VLSR	e	Tel.V
id	yr	mas	mas	mas	mas	yr	yr	km/s	km/s	yr
1	1992.224	-6.4	4.6	172.0	4.7	NTT	2000.487	1199	100	Keck
2	1994.314	-28.5	4.8	179.0	3.4	NTT	2002.418	-495	40	Keck
3	1995.534	-37.3	3.8	172.1	4.3	NTT	2002.421	-530	45	Keck
4	1996.253	-43.4	3.6	164.4	3.6	NTT	2003.438	-1550	22	Keck
5	1996.427	-45.9	1.9	161.5	5.3	NTT	2004.474	-1143	57	Keck
6	1997.544	-59.0	3.4	130.4	2.8	NTT	2005.410	-926	16	Keck
7	1998.373	-65.3	4.7	122.1	3.5	NTT	2005.504	-850	31	Keck
8	1999.465	-67.5	4.4	106.0	4.1	NTT	2006.391	-692	21	Keck
9	2000.472	-55.3	5.0	63.9	3.1	NTT	2006.461	-718	17	Keck
10	2000.523	-62.8	3.1	57.6	3.1	GEMINI	2006.495	-695	36	Keck
11	2001.502	-49.3	3.8	23.8	2.1	NTT	2006.497	-713	26	Keck
12	2002.488	-27.8	11.5	14.9	10.4	NTT	2007.385	-483	50	Keck
13	2002.250	-3.1	4.0	-6.6	4.0	VLT	2007.548	-506	50	Keck
14	2002.335	6.6	2.7	-7.6	2.7	VLT	2003.353	-1512	49	VLT
15	2002.393	16.3	3.8	0.0	3.8	VLT	2003.446	-1428	63	VLT
16	2002.409	18.2	3.3	2.1	3.3	VLT	2003.271	-1571	52	VLT
17	2002.412	17.3	3.3	2.3	3.3	VLT	2004.535	-1055	41	VLT
18	2002.414	17.4	3.3	3.2	3.3	VLT	2004.537	-1056	33	VLT
19	2002.578	30.8	3.3	20.7	3.3	VLT	2004.632	-1039	34	VLT
20	2002.660	34.1	3.2	26.9	3.2	VLT	2005.158	-1001	68	VLT
21	2002.660	33.7	3.2	27.3	3.2	VLT	2005.212	-960	33	VLT
22	2003.214	41.1	0.3	66.6	0.4	VLT	2005.215	-910	48	VLT
23	2003.351	41.4	0.3	75.0	0.3	VLT	2005.455	-839	53	VLT
24	2003.356	40.7	0.4	74.8	0.4	VLT	2005.461	-907	38	VLT
25	2003.446	40.6	0.5	79.8	0.5	VLT	2005.677	-774	68	VLT
26	2003.451	41.3	0.4	80.4	0.4	VLT	2005.769	-860	51	VLT
27	2003.452	41.5	0.3	80.5	0.3	VLT	2006.204	-702	37	VLT
28	2003.454	40.9	0.3	80.6	0.3	VLT	2006.305	-718	68	VLT
29	2003.454	41.3	0.4	81.8	0.4	VLT	2006.624	-658	50	VLT
30	2003.550	40.9	0.3	85.3	0.3	VLT	2007.230	-586	50	VLT
31	2003.676	40.6	0.3	91.8	0.3	VLT	2007.304	-537	50	VLT
32	2003.678	41.1	0.6	91.6	0.6	VLT	2007.550	-505	50	VLT
33	2003.761	40.2	0.4	96.6	0.4	VLT	2007.673	-482	50	VLT
34	2004.240	37.2	0.9	113.2	0.9	VLT	2008.262	-394	24	VLT
35	2004.325	36.9	0.3	116.2	0.3	VLT	2008.431	-425	55	VLT
36	2004.347	36.1	0.3	117.6	0.3	VLT	2009.385	-272	40	VLT
37	2004.443	36.0	0.3	120.4	0.3	VLT				
38	2004.513	35.3	0.3	123.1	0.3	VLT				
39	2004.516	35.2	0.6	123.1	0.6	VLT				
40	2004.574	34.4	0.4	124.8	0.4	VLT				
41	2004.574	34.3	0.6	123.9	0.6	VLT				
42	2004.664	33.6	0.3	127.2	0.3	VLT				
43	2004.730	34.0	0.7	128.9	0.7	VLT				
44	2005.270	28.1	0.3	143.0	0.3	VLT				
45	2005.366	27.0	0.3	145.2	0.3	VLT				
46	2005.467	26.3	0.4	146.9	0.4	VLT				
47	2005.576	24.9	0.4	149.4	0.4	VLT				
48	2006.324	17.5	0.8	161.7	0.6	VLT				
49	2007.545	2.8	0.9	175.7	0.7	VLT				
50	2007.550	4.1	0.4	175.2	0.4	VLT				

Result truncated to 50 rows

plot the output query using TAP:SQL



Tap VizieR

2

Warning: Crossmatch tuning note

The TAPVizieR service provides VizieR tables using ADQL (a SQL extension in Astronomy).

Type your ADQL Query in the bottom area or try an example -- or use the VizieR capabilities to construct your ADQL query

Search tables %20JApJ/707/L114/table1 Go

Search by catalog, author's name, word(s) from title, position (resolved by Sesame), ...
e.g. : Veron, 2Mass, redshift, M31...

all by wavelength by mission by astronomy x

Show 10 entries

catalogues	description	tables
JApJ/707/L114	{vizContent{timeSerie} wavelength : optical ; astrometry : Positional_Data ; Multiple_Stars ; Velocities ;	☒ JApJ/707/L114/table1 (96 rows) Orbital data for S2 used in this work Orbital data for S2 used in this work

Showing 1 to 1 of 1 entries

Columns and constraints Sky area Join Unit change of coordinates

Modify the query with the upper tabs.

Max records ☐ all ☒ limit 100 Update query

Preuzimanje podataka

<https://vizier.cds.unistra.fr/viz-bin/VizieR>

allby wavelengthby missionby astronomy✕

Show 10 entries

catalogues	description	tables
J/ApJ/707/L114	\vizContent{timeSerie} wavelength : optical ; astronomy : Positional_Data ; Multiple_Stars ; Velocities ;	☐ J/ApJ/707/L114/table1 (96 rows) Orbital data for S2 used in this work Orbital data for S2 used in this work

Showing 1 to 1 of 1 entries

Construct your query

Upload your data

Name File/Url

* to use an uploaded table in the query, you must prefix its name with TAP_UPLOAD (i.e. TAP_UPLOAD.myTable).

✕Columns and constraints

* Modify the query with the upper tabs.

Max records ☐ all ☒ limit 100

Update query

```
1 -- output format : csv
2 SELECT TOP 100 "J/ApJ/707/L114/table1".recno, "J/ApJ/707/L114/table1"."Ep.A", "J/ApJ/707/L114/table1".oRA, "J/ApJ/707/L114/table1".e_oRA, "J/ApJ/707/L114/table1".oDE, "J/ApJ/707/L114/table1".e_oDE, "J/ApJ/707/L114/table1"."Tel.A",
3 "J/ApJ/707/L114/table1"."Ep.V", "J/ApJ/707/L114/table1".VLSR, "J/ApJ/707/L114/table1".e_VLSR, "J/ApJ/707/L114/table1"."Tel.V"
4 FROM "J/ApJ/707/L114/table1"
```

Query name "J/ApJ/707/L114/table1" Output format csv

Run

Quickview

Reset

Test

List of your TAP queries

Refresh

Abort

Destroy

Properties

Show all entries

name	phase	start	destruction	results
'J/ApJ/707/L114/table1'	COMPLETED	Mon Nov 17 22:58:44 CET 2025	Wed Nov 19 22:58:44 CET 2025	download (csv)

Showing 1 to 1 of 1 entries

Preuzimanje podataka

<https://vizier.cds.unistra.fr/viz-bin/VizieR>

Query name Output format **Run** Quickview Reset Test

List of your TAP queries Refresh Abort Destroy Properties

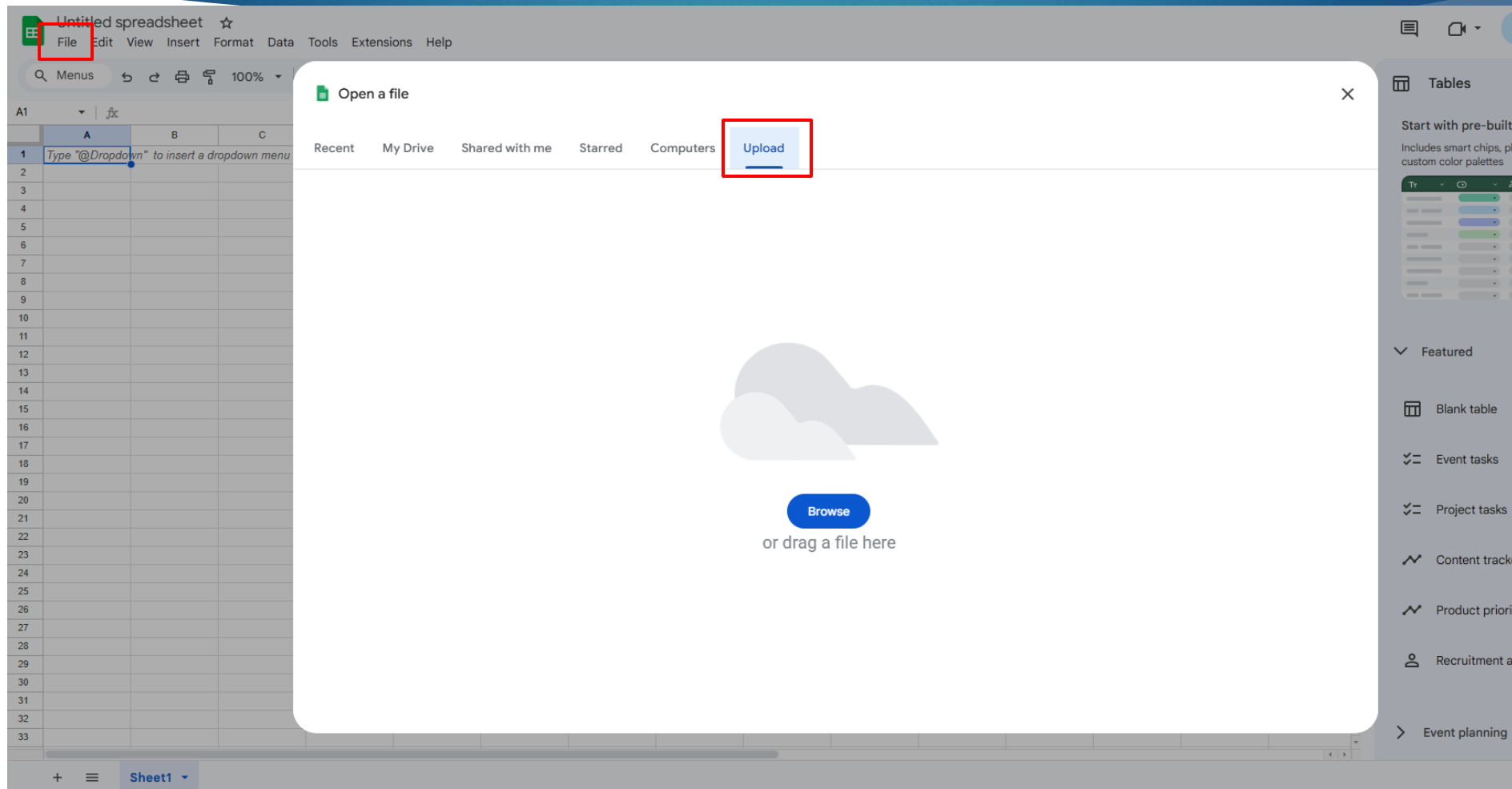
Show entries Search:

name	phase	start	destruction	results
'J/ApJ/707/L114/table1'	COMPLETED	Tue Nov 18 16:32:41 CET 2025	Thu Nov 20 16:32:41 CET 2025	download (csv)
'J/ApJ/707/L114/table1'	COMPLETED	Mon Nov 17 22:58:44 CET 2025	Wed Nov 19 22:58:44 CET 2025	download (csv)

Showing 1 to 2 of 2 entries First Previous Next Last

Obrada podataka

<https://www.geogebra.org/calculator>



Obrada podataka

<https://www.geogebra.org/calculator>

S2 star - org ☆ 📁 ☁

File Edit View Insert Format Data Tools Extensions Help

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A1 ▾ | fx recno

	A	B	C	D	E	F	G	H	I	J	K	L
1	recno	Ep_A	oRA	e_oRA	oDE	e_oDE	Tel_A	Ep_V	VLSR	e_VLSR	Tel_V	
2	1	1992.224	-6.4	4.6	172	4.7	NTT	2000.487	1199	100	Keck	
3	2	1994.314	-28.5	4.8	179	3.4	NTT	2002.418	-495	40	Keck	
4	3	1995.534	-37.3	3.8	172.1	4.3	NTT	2002.421	-530	45	Keck	
5	4	1996.253	-43.4	3.6	164.4	3.6	NTT	2003.438	-1550	22	Keck	
6	5	1996.427	-45.9	1.9	161.5	5.3	NTT	2004.474	-1143	57	Keck	
7	6	1997.544	-59	3.4	130.4	2.8	NTT	2005.41	-926	16	Keck	
8	7	1998.373	-65.3	4.7	122.1	3.5	NTT	2005.504	-850	31	Keck	
9	8	1999.465	-67.5	4.4	106	4.1	NTT	2006.391	-692	21	Keck	
10	9	2000.472	-55.3	5	63.9	3.1	NTT	2006.461	-718	17	Keck	
11	10	2000.523	-62.8	3.1	57.6	3.1	GEMINI	2006.495	-695	36	Keck	
12	11	2001.502	-49.3	3.8	23.8	2.1	NTT	2006.497	-713	26	Keck	
13	12	2002.488	27.8	11.5	14.9	10.4	NTT	2007.385	-483	50	Keck	
14	13	2002.25	-3.1	4	-6.6	4	VLT	2007.548	-506	50	Keck	
15	14	2002.335	6.6	2.7	-7.6	2.7	VLT	2003.353	-1512	49	VLT	
16	15	2002.393	16.3	3.8	0	3.8	VLT	2003.446	-1428	63	VLT	
17	16	2002.409	18.2	3.3	2.1	3.3	VLT	2003.271	-1571	52	VLT	
18	17	2002.412	17.3	3.3	2.3	3.3	VLT	2004.535	-1055	41	VLT	
19	18	2002.414	17.4	3.3	3.2	3.3	VLT	2004.537	-1056	33	VLT	
20	19	2002.578	30.8	3.3	20.7	3.3	VLT	2004.632	-1039	34	VLT	
21	20	2002.66	34.1	3.2	26.9	3.2	VLT	2005.158	-1001	68	VLT	
22	21	2002.66	33.7	3.2	27.3	3.2	VLT	2005.212	-960	33	VLT	
23	22	2003.214	41.1	0.3	66.6	0.4	VLT	2005.215	-910	48	VLT	
24	23	2003.351	41.4	0.3	75	0.3	VLT	2005.455	-839	53	VLT	
25	24	2003.356	40.7	0.4	74.8	0.4	VLT	2005.461	-907	38	VLT	
26	25	2003.446	40.6	0.5	79.8	0.5	VLT	2005.677	-774	68	VLT	
27	26	2003.451	41.3	0.4	80.4	0.4	VLT	2005.769	-860	51	VLT	
28	27	2003.452	41.5	0.3	80.5	0.3	VLT	2006.204	-702	37	VLT	
29	28	2003.452	41.5	0.3	80.5	0.3	VLT	2006.204	-702	37	VLT	

Obrada podataka

<https://www.geogebra.org/calculator>

S2 star ☆ 📁 Saved to Drive

File Edit View Insert Format Data Tools Extensions Help

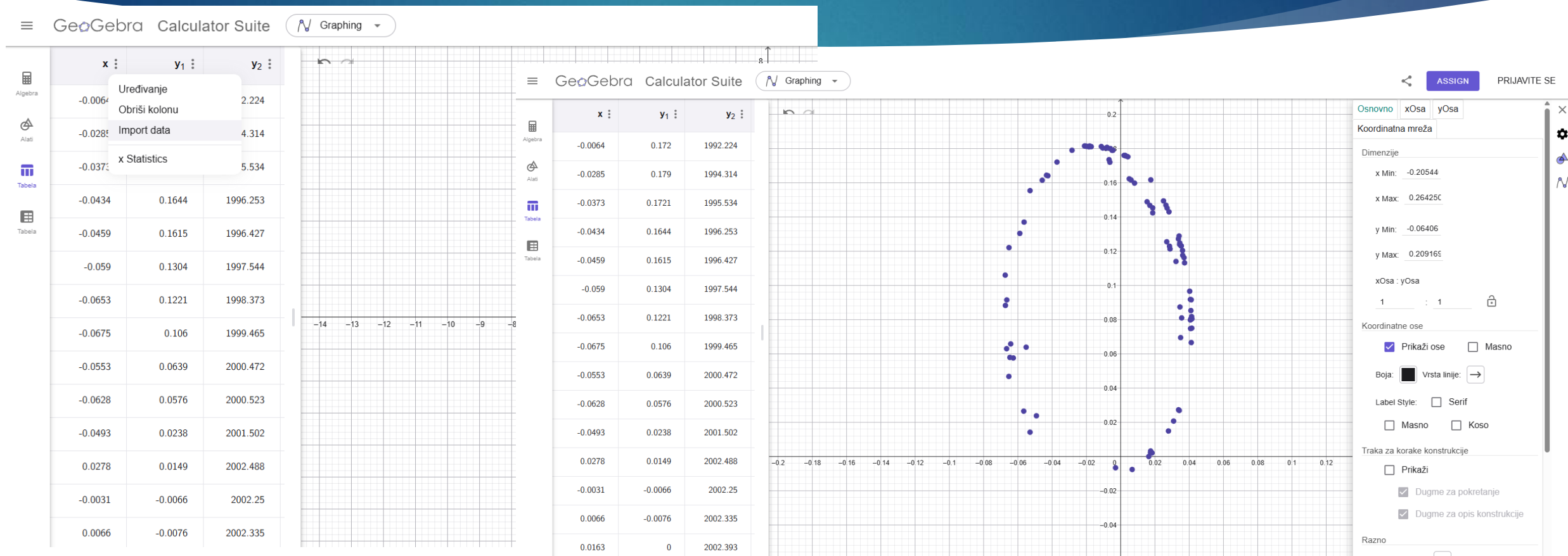
Q Menus ↶ ↷ 🖨 📄 100% | \$ % .0+ .00 123 | Defaul... ▾ | -

A1 ▾ | $\int$ oRA [as]

	A	B	C	D	E	F
1	oRA [as]	oDE [as]	Ep_A			
2	-0.0064	0.172	1992.224			
3	-0.0285	0.179	1994.314			
4	-0.0373	0.1721	1995.534			
5	-0.0434	0.1644	1996.253			
6	-0.0459	0.1615	1996.427			
7	-0.059	0.1304	1997.544			
8	-0.0653	0.1221	1998.373			
9	-0.0675	0.106	1999.465			
10	-0.0553	0.0639	2000.472			
11	-0.0628	0.0576	2000.523			
12	-0.0493	0.0238	2001.502			
13	0.0278	0.0149	2002.488			
14	-0.0031	-0.0066	2002.25			
15	0.0066	-0.0076	2002.335			
16	0.0163	0	2002.393			
17	0.0182	0.0021	2002.409			
18	0.0173	0.0023	2002.412			
19	0.0174	0.0032	2002.414			
20	0.0308	0.0207	2002.578			
21	0.0341	0.0269	2002.66			
22	0.0337	0.0273	2002.66			
23	0.0411	0.0666	2003.214			
24	0.0414	0.075	2003.351			
25	0.0407	0.0748	2003.356			

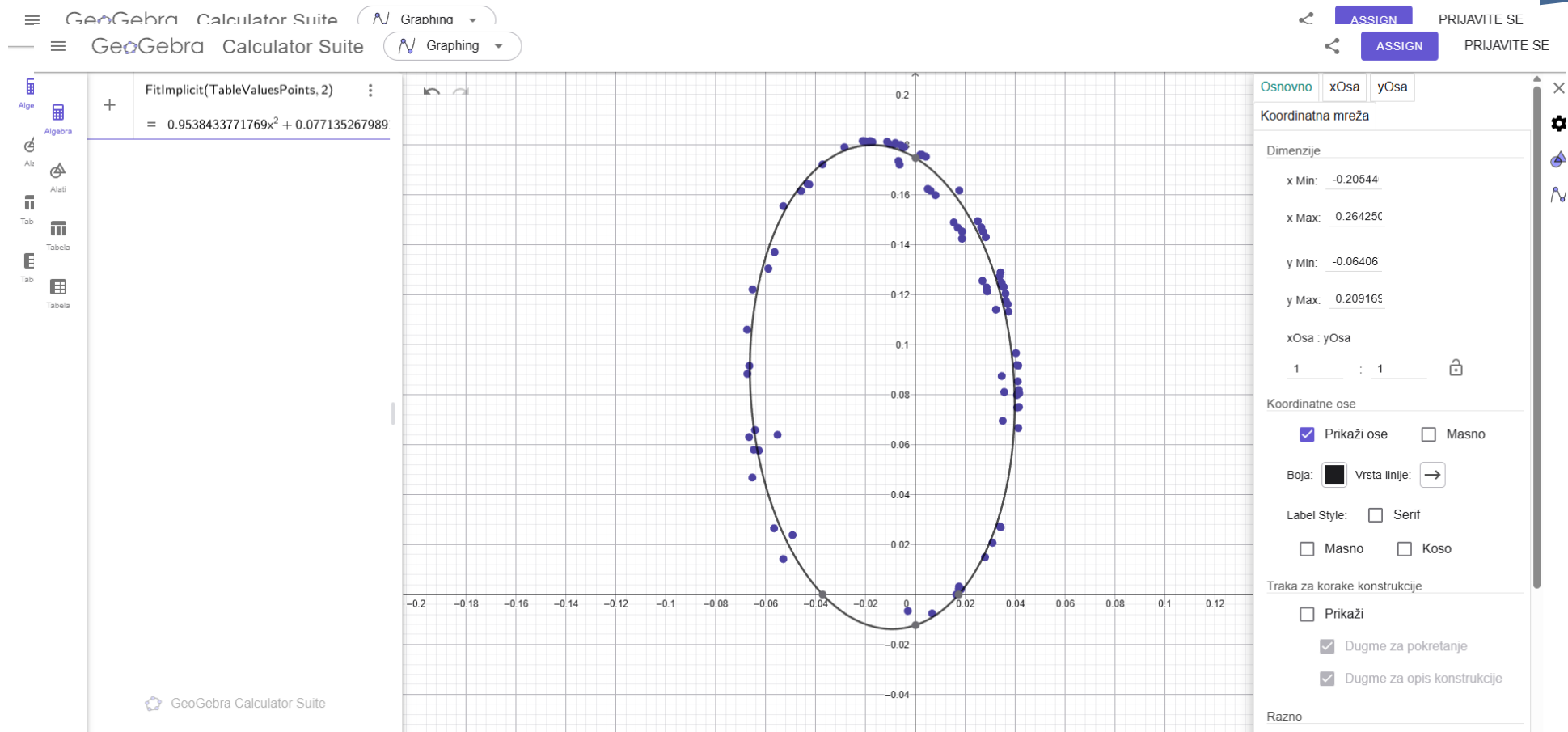
Obrada podataka

<https://www.geogebra.org/calculator>



Obrada podataka

<https://www.geogebra.org/calculator>



Obrada podataka

<https://www.geogebra.org/calculator>

► Jednačina: $0.95x^2 + 0.08xy + 0.02x + 0.29y^2 - 0.05y = 0$

► Centar elipse:

$$2Ax_0 + By_0 + D = 0$$

$$Bx_0 + 2Cy_0 + E = 0$$

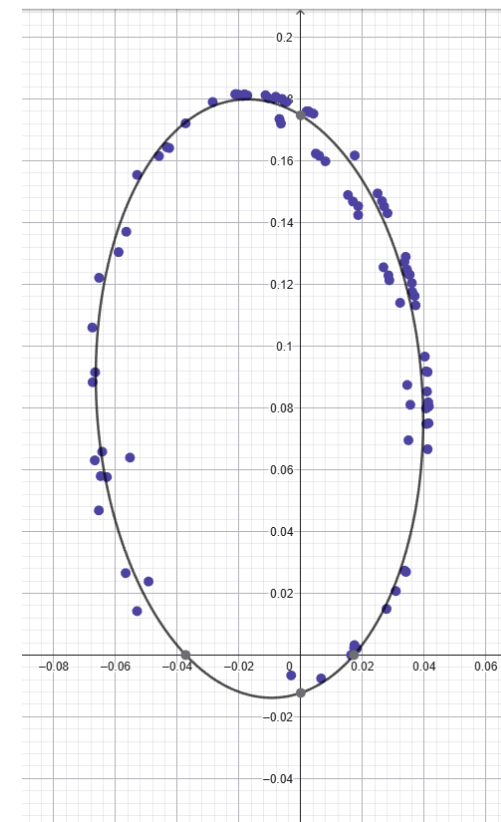
► Rešenje:

$$x_0 \approx -0.014$$

$$y_0 \approx 0.088$$

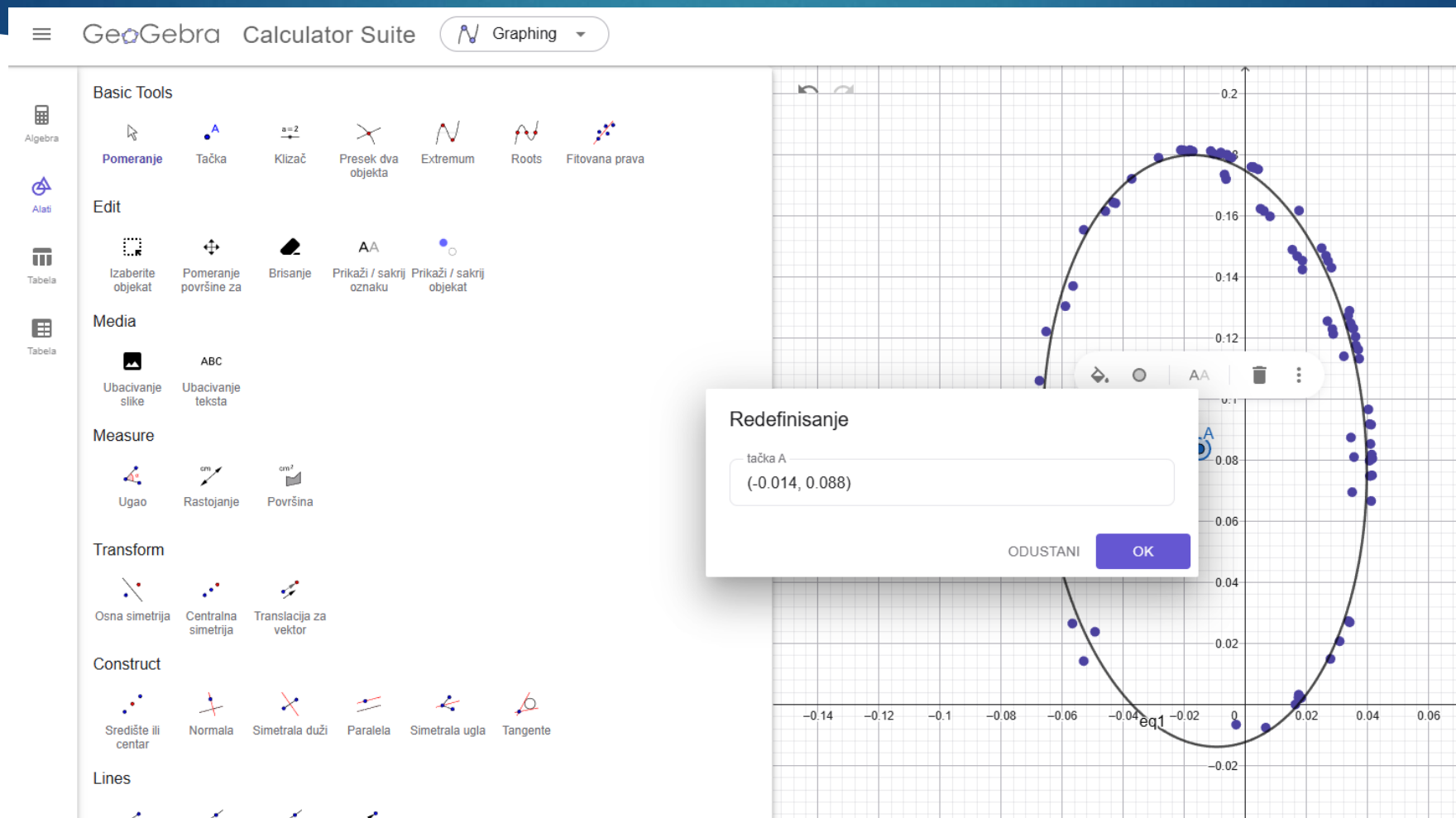
► Nagib elipse:

$$\theta = \frac{1}{2} \arctan \frac{B}{A - C} \approx 0.06 \approx 3.5^\circ$$



Obrada podataka

<https://www.geogebra.org/calculator>



Obrada podataka

<https://www.geogebra.org/calculator>

GeoGebra Calculator Suite Graphing

Global

Jezik: Serbian / српски

Zaokrugljivanje: 10 decimal. mesta

Označavanje: Za sve nove objekte

Veličina slova: 16 pt

SNIMI PODEŠAVANJE

RESTORE SETTINGS

eq1: FitImplicit(TableValuesPoints, 2)
 $= 0.9538433772x^2 + 0.077135268x y + 0.0190757931x + 0.2858556505y^2 - 0.0464246428$

A = (-0.014, 0.088)

B = (-0.07, 0.12)

C = (-0.05, 0.16)

t1 = Mnogougao(A, B, C)
 $= 0.00144$

a = Duž(B, C, t1)
 $= 0.0447213595$

b = Duž(C, A, t1)
 $= 0.0804984472$

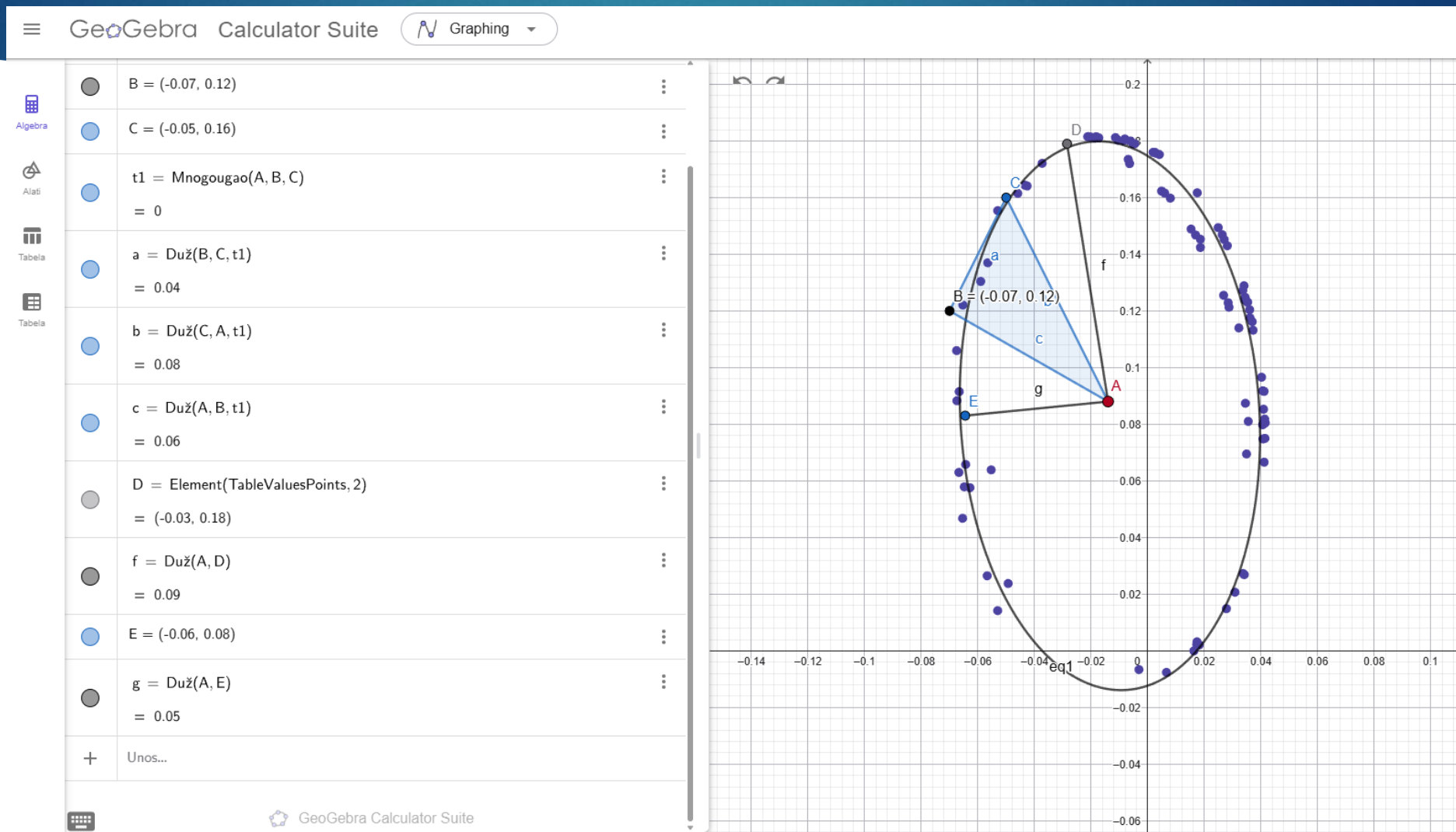
c = Duž(A, B, t1)
 $= 0.064498062$

Unos...

The graph shows a circle with a grid. Points A, B, and C are marked on the circle. A triangle ABC is formed by these points. The calculator displays the implicit equation of the circle and various geometric calculations.

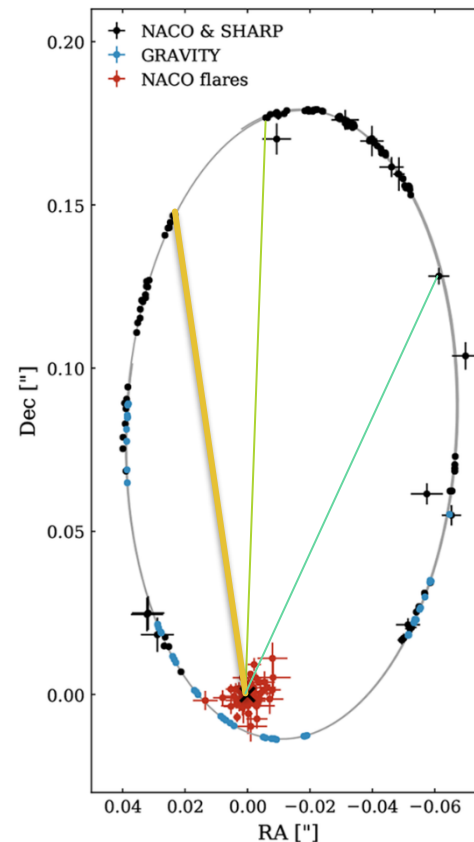
Obrada podataka

<https://www.geogebra.org/calculator>



Vežba – izračunajte masu SMBH

- ▶ ...
- ▶ Odrediti period (P)
 - ▶ $A_{ell} = ab \cdot \pi$ - a i b sa slike, prethodni slajd
 - ▶ $\Delta A = \frac{\Delta t}{P} \cdot A_{ell}$ - II Keplerov zakon
 - ▶ Nepoznato ΔA , Δt , A_{ell}
 - ▶ ΔA i Δt – sa slike (prethodni slajd, za svaki segment)
 - ▶ Nacrtati trougao i odrediti njegovu površinu (ponoviti više puta!)
 - ▶ Izračunati masu SMBH
 - ▶ III Keplerov zakon
 - ▶ $P^2 = \frac{4\pi^2}{G(M+m_{S2})} a^3$, $M \gg m_{S2}$



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$$\varepsilon = \sqrt{1 - \frac{b^2}{a^2}}$$
$$\frac{dA}{dt} = \frac{abn}{2}$$
$$\frac{a^3}{T^2} = M$$