



Exponentially Fast

From Ptolemy via Friedmann to Cosmic Inflation

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The Outline of the Talk

1. Old questions, old pictures
2. Friedmann: the Universe in two equations
3. The hot Big Bang and its evidence
4. Cracks in the picture
5. Cosmic inflation
6. Our research: tachyon, RS and holographic inflation
7. Where we are today

Three questions as old as humanity

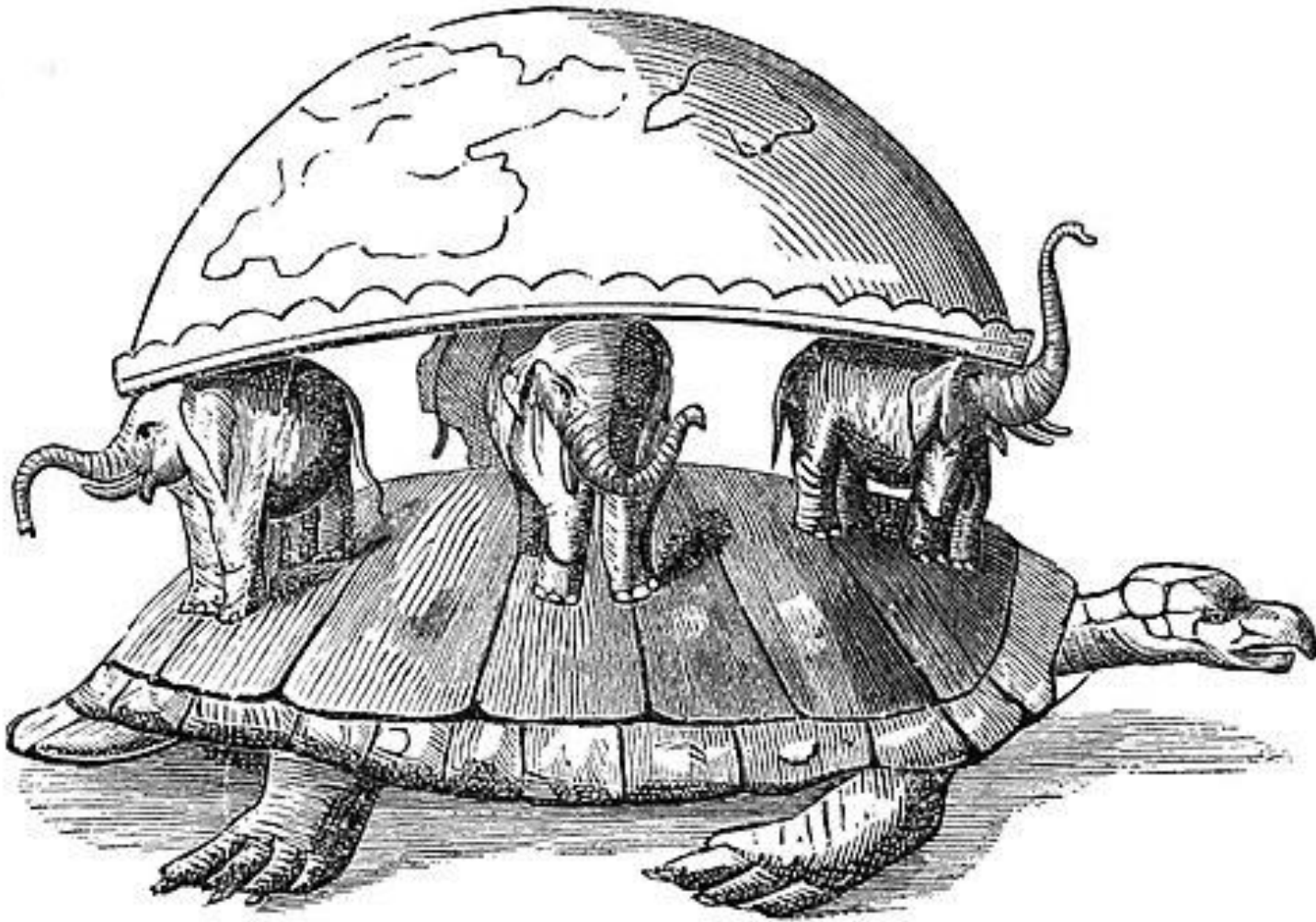
How did the Universe **begin**?

How has it **evolved**?

Where is it **heading**?

- Cosmology: physics and history of the Universe as a whole.
- Astronomy is a *time machine*: light needs time to reach us. We see the Sun as it was 8 minutes ago, Andromeda as it was 2.5 million years ago.
- Records: GN-z11 (Hubble, 2016), ≈ 400 Myr after the Big Bang; MoM-z14 (JWST, 2025), redshift $z = 14.44$, only ≈ 280 Myr after the Big Bang.

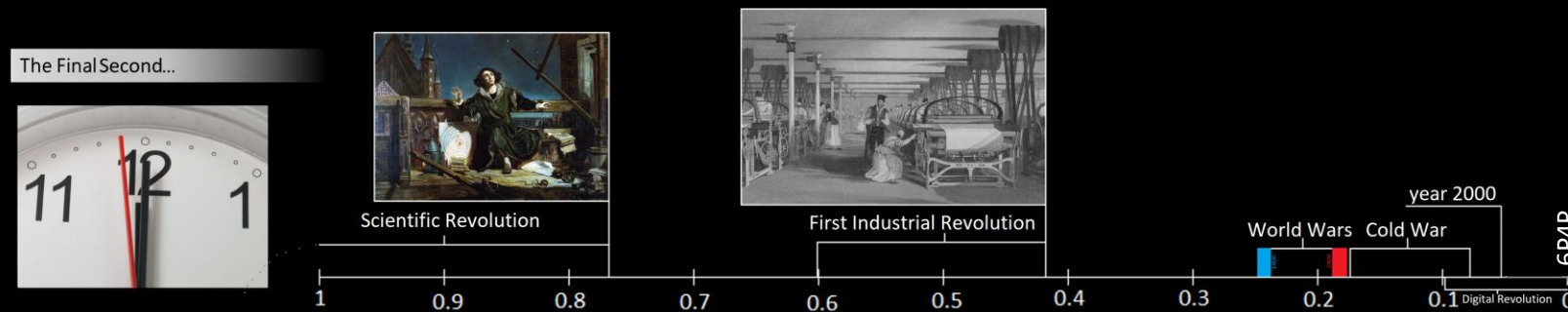
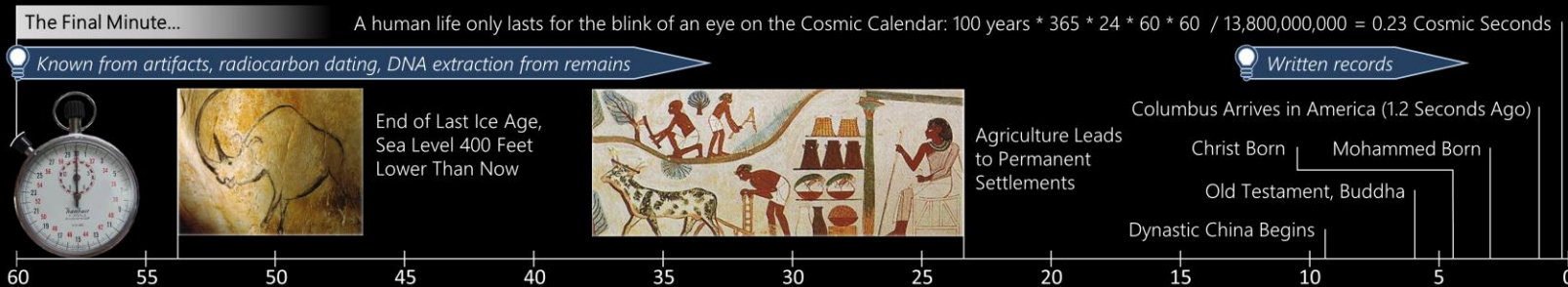
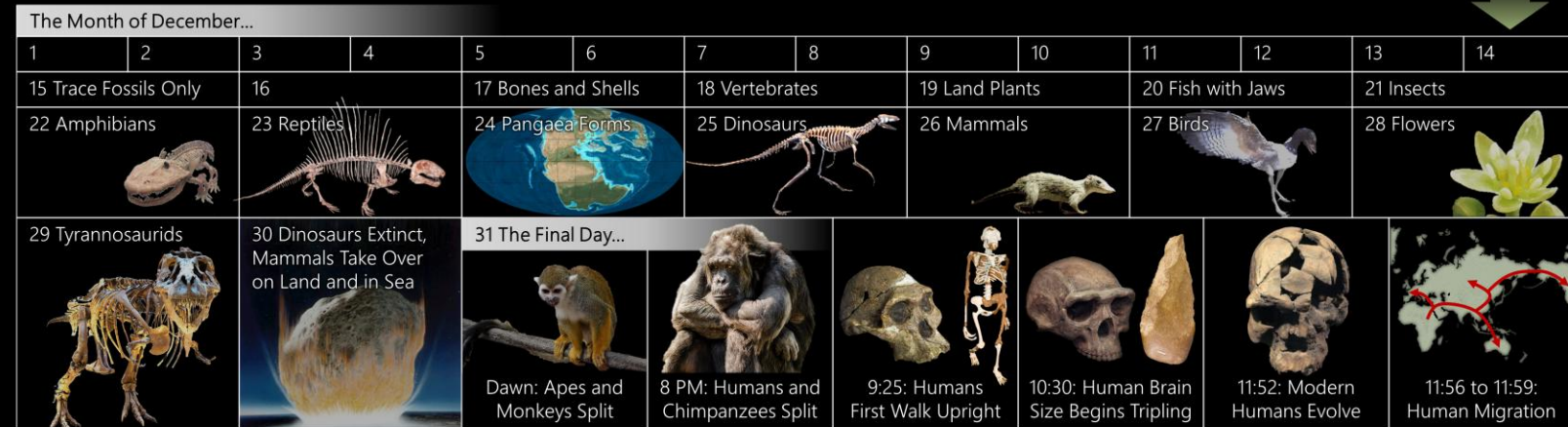
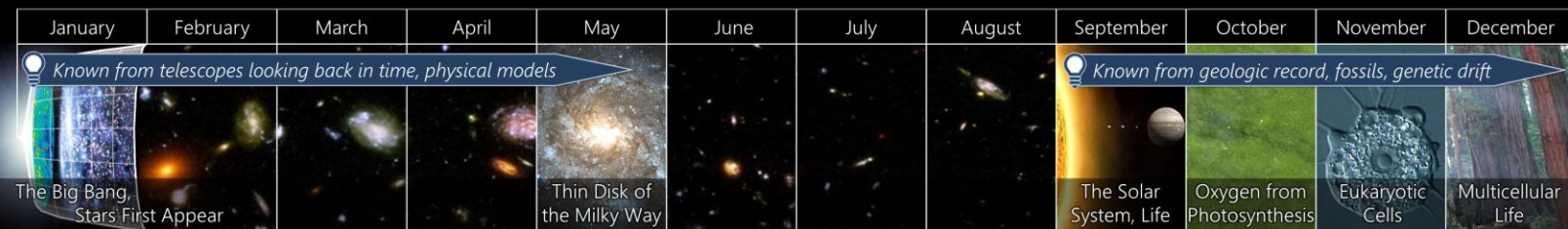
Maybe...? ☺



If the history of the Universe is one day...

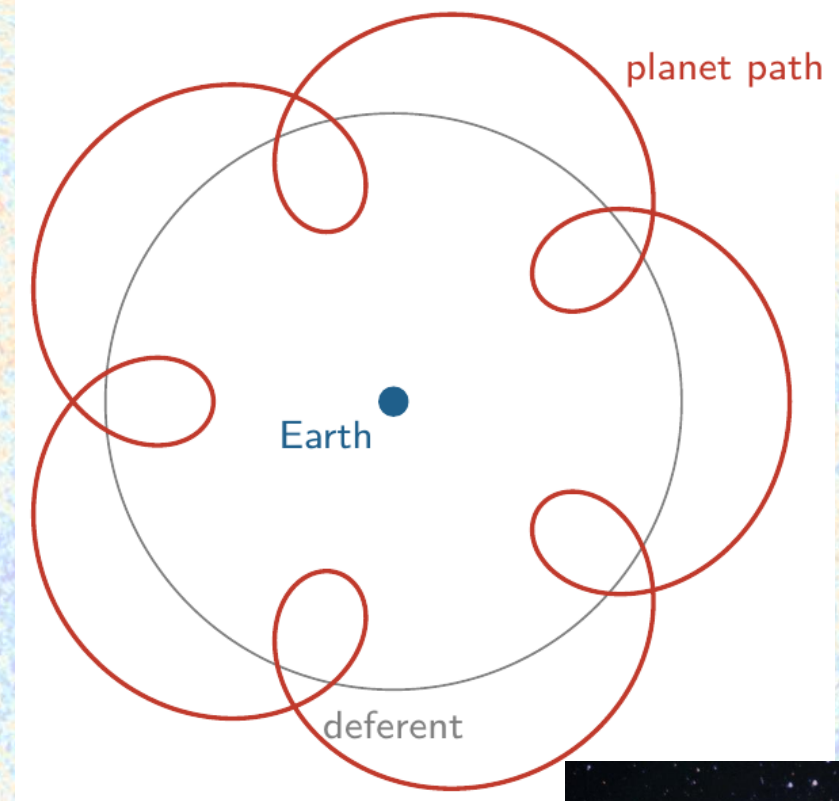
The Cosmic Calendar

The 13.8 billion year history of the universe scaled down to a single year, where the Big Bang is January 1st at midnight, and right now is midnight 1 year later



Ptolemy: the first “cosmological model”

- Aristotle (4th c. BC): the Earth is a sphere (Moon-eclipse shadow, star heights, ships' sails) - but static and central.
- Ptolemy (2nd c. AD): nested crystal spheres, Earth in the centre.
- Planets on **epicycles** on *deferents* reproduce retrograde motion.
- Describes *what* the sky does - no origin, no past, no future.

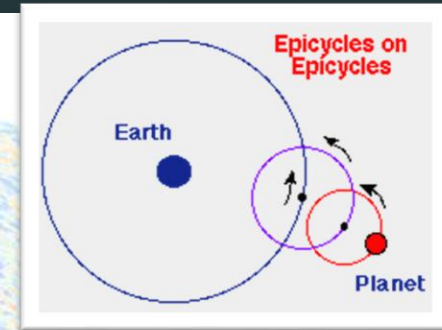


Lesson: a model can fit the data for 1400 years and still be the wrong picture.



Nicolaus Copernicus (1473 – 1543)

- Book „*De revolutionibus orbium coelestium*“ (*On the Revolutions of the Celestial Spheres*)



40 circles



Ptolemej (Almagest)

Some sources state that Ptolemy's model had only 30 circles, while others claim it had 80 (a myth!).

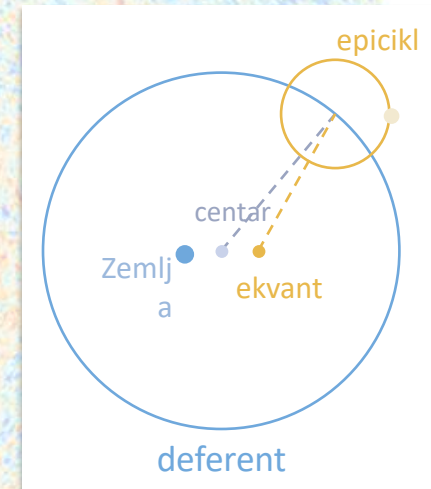
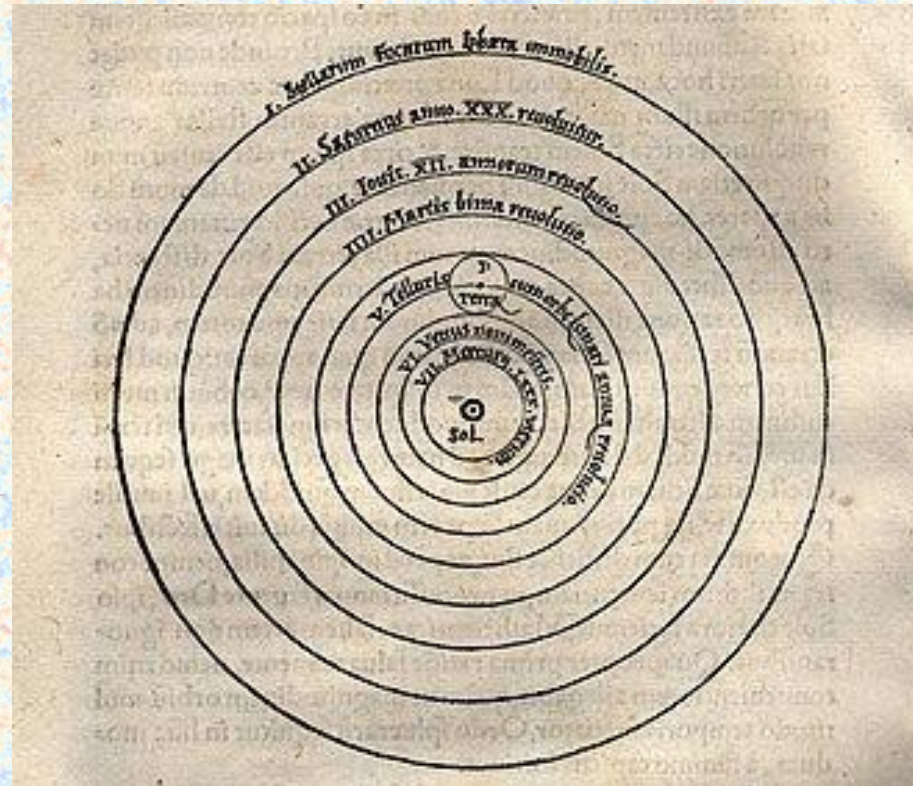
48 circles



Kopernik (1543)

Initially, 34 circular motions, later expanded.

Circle $\neq$ epicycle

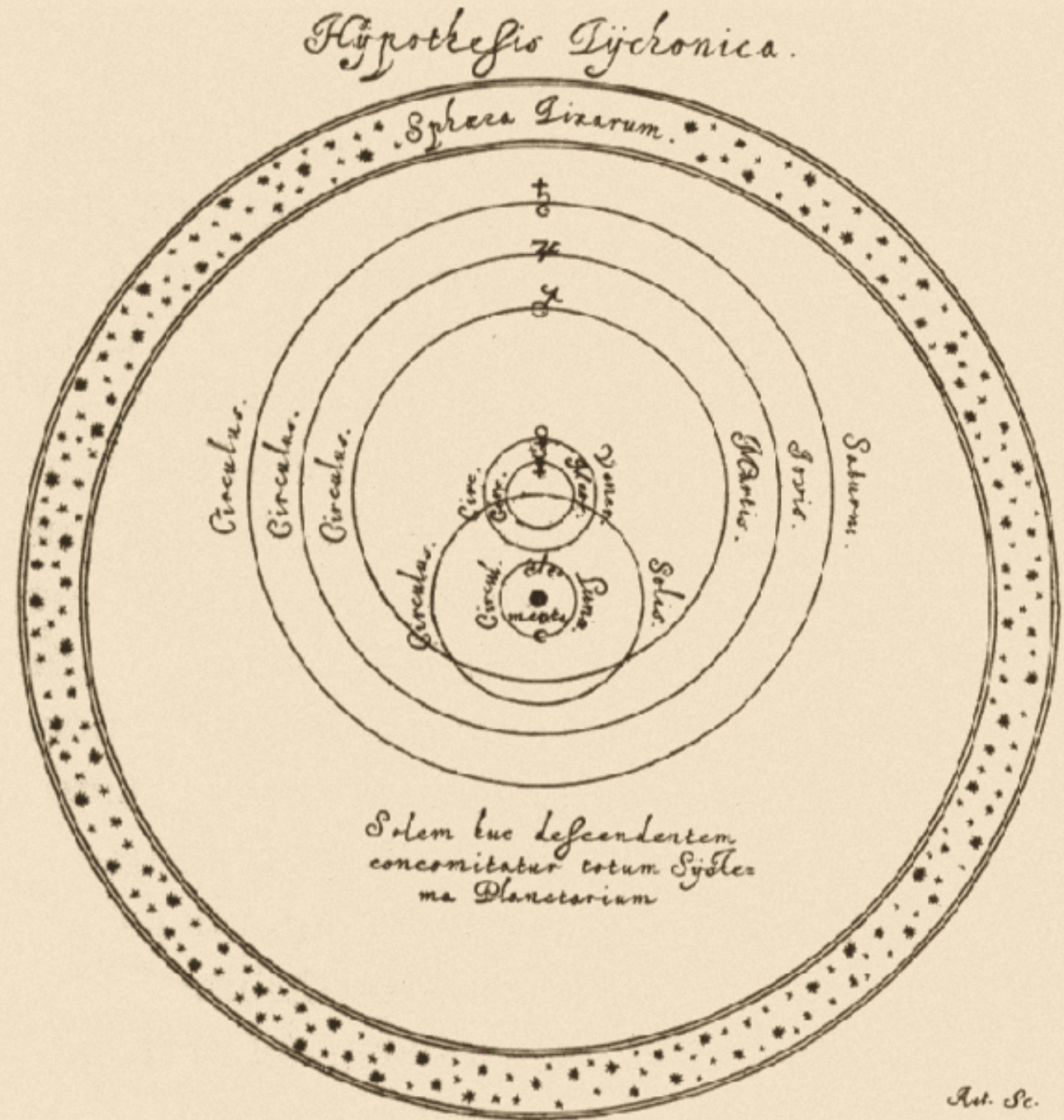


Tycho Brahe (1546 – 1601)

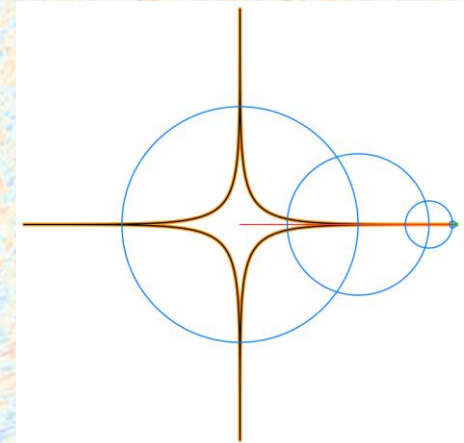
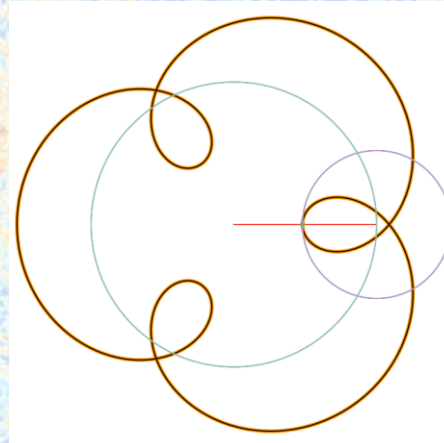
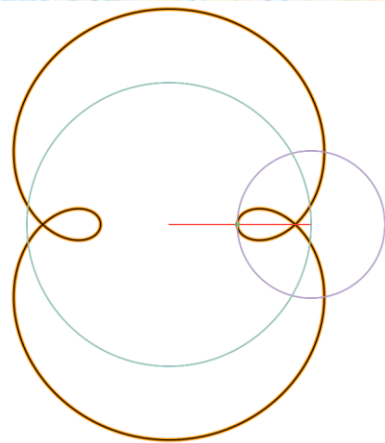
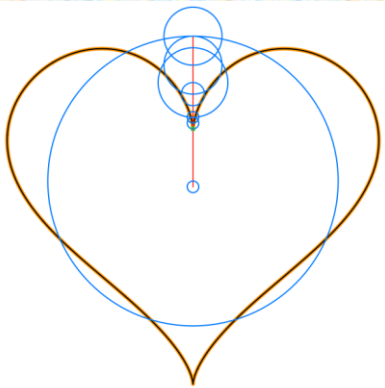
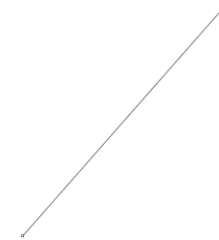
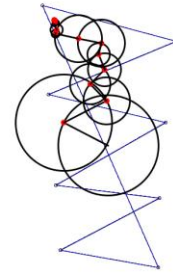
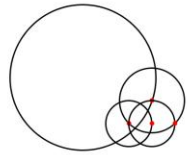
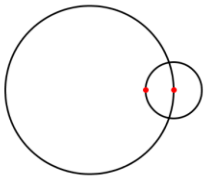
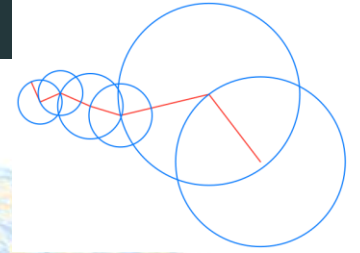


Portrait(1596), oil on canvas

A 17th century illustration of the Hypothesis Tychonica, from Hevelius' Selenographia, 1647
page 163



Fourier series expansion



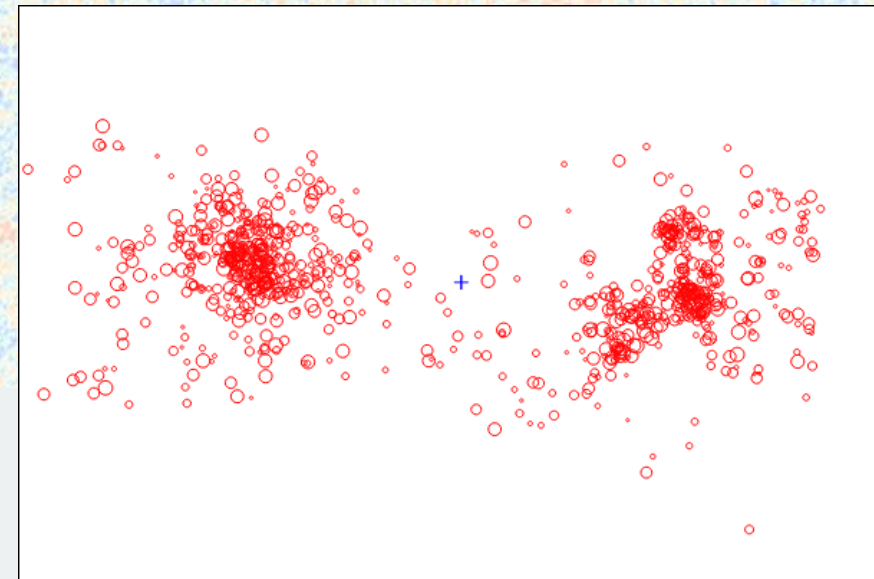
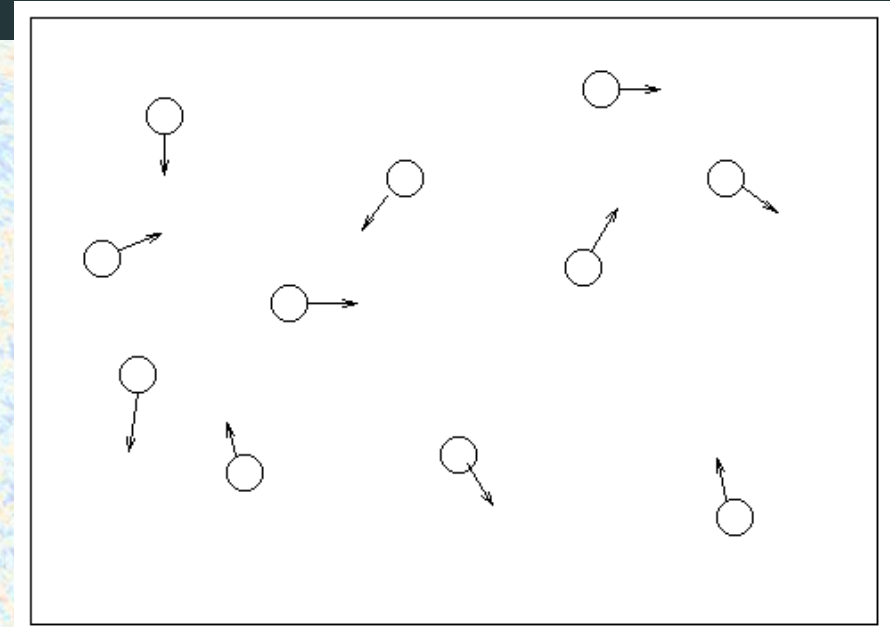
- M. Lopez, [*The Epicycles of Ancient Astronomy*](#)
- [*Epicycles - Experiment with complex Fourier series*](#)

Copernicus, Kepler, Newton ... and a worry

- Copernicus (1543): Sun in the „centre“. Kepler: ellipses. Galileo: telescope. Newton: **gravity**.
- Stars are other Suns. Newton's dilemma: **gravity is attractive, so why doesn't everything collapse?**
 - finite number of stars $\Rightarrow$ collapse toward the centre;
 - infinite, uniform distribution $\Rightarrow$ “balanced” (an unstable balance, as we now know).
- Until the 20th century: the Universe is eternal, static, unchanging.

A one-line puzzle with a deep answer

If the Universe is infinite and eternal, *why is the night sky dark?*



Olbers' paradox - a textbook computation

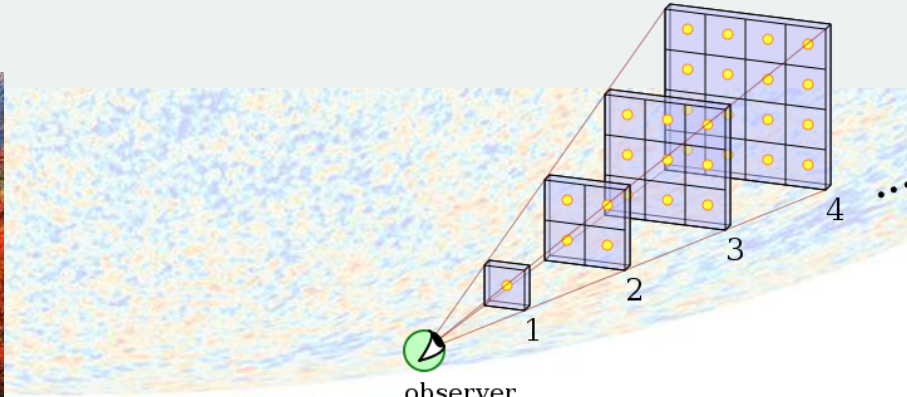
Uniform stars, density n , luminosity L . Take a shell of radius r and thickness dr :

$$\underbrace{dN = 4\pi r^2 n dr}_{\text{stars in shell}} \times \underbrace{F = \frac{L}{4\pi r^2}}_{\text{flux of one star}} \Rightarrow dF_{\text{shell}} = nL dr \quad (\text{independent of } r!)$$

Every shell contributes equally, so $F_{\text{total}} = nL \int_0^\infty dr = \infty$ — the sky should blaze like a stellar surface. (Nearer stars hide farther ones, but the sky is then *covered* with stellar discs: $\sim 5\,000$ K everywhere.)

Resolution

The Universe has a **finite age**: we only see stars within $r \lesssim c t_0$. Light from farther away has not arrived yet - expansion (redshift) also dims it. The dark night sky is the first observational evidence of a Universe with a history.



Einstein 1915: geometry is gravity

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{\text{geometry}} = \frac{8\pi G}{c^4} \underbrace{T_{\mu\nu}}_{\text{matter, energy}}$$

- “Matter tells spacetime how to curve; spacetime tells matter how to move.” (J. A. Wheeler)
- 1917: Einstein applies it to the whole Universe - and finds it *cannot* be static.
- He adds a constant:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

The Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$

“My biggest blunder”?

Λ , invented to *stop* the Universe, is today's **dark energy** - 68% of the cosmic budget.

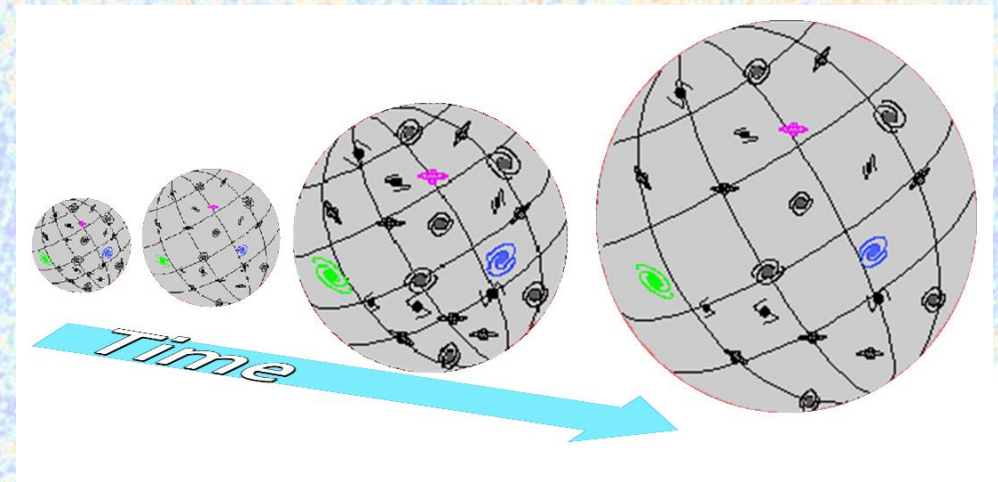


Friedmann, Lemaître and an expanding space

Assume the Universe is *homogeneous and isotropic* (same everywhere, same in all directions - true on scales $\gtrsim 100$ Mpc). Exact solution of Einstein's equations (Friedmann 1922; Lemaître 1927; Robertson 1935, Walker 1937):

$$ds^2 = c^2 dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

- $a(t)$: **scale factor**. $k = 0, \pm 1$: curvature.
- Galaxies keep their comoving coordinates $\vec{x}$; the distances stretch: $\vec{r}_{\text{phys}} = a(t) \vec{x}$.



Dots on a balloon: no centre, everything recedes from everything.

Hubble–Lemaître law and the distance ladder

Redshift $1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} = \frac{a_0}{a_{\text{emit}}}$. For nearby galaxies

$$v = H_0 D, \quad H_0 = \left. \frac{\dot{a}}{a} \right|_{\text{today}}$$

- Lemaître 1927, Hubble 1929:

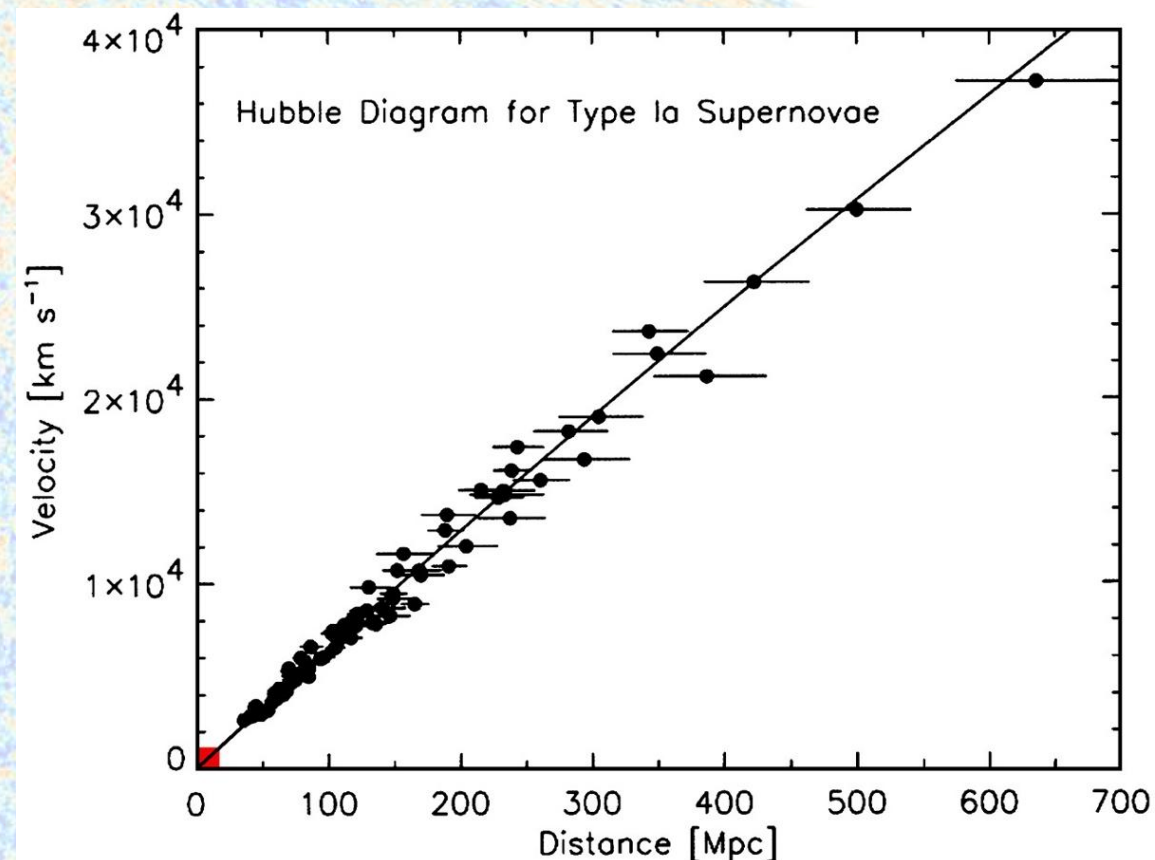
$$H_0 \approx 500 \text{ km s}^{-1} \text{ Mpc}^{-1} (!).$$

Estimate: age of the Universe

$t_H = 1/H_0$. With $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$

$$t_H = \frac{3.09 \times 10^{19}}{70} \text{ s} \approx 4.4 \times 10^{17} \text{ s} \approx 14 \text{ Gyr}$$

Hubble's 500: only 2 Gyr — younger than the Earth!
(Cepheid calibration was off.)

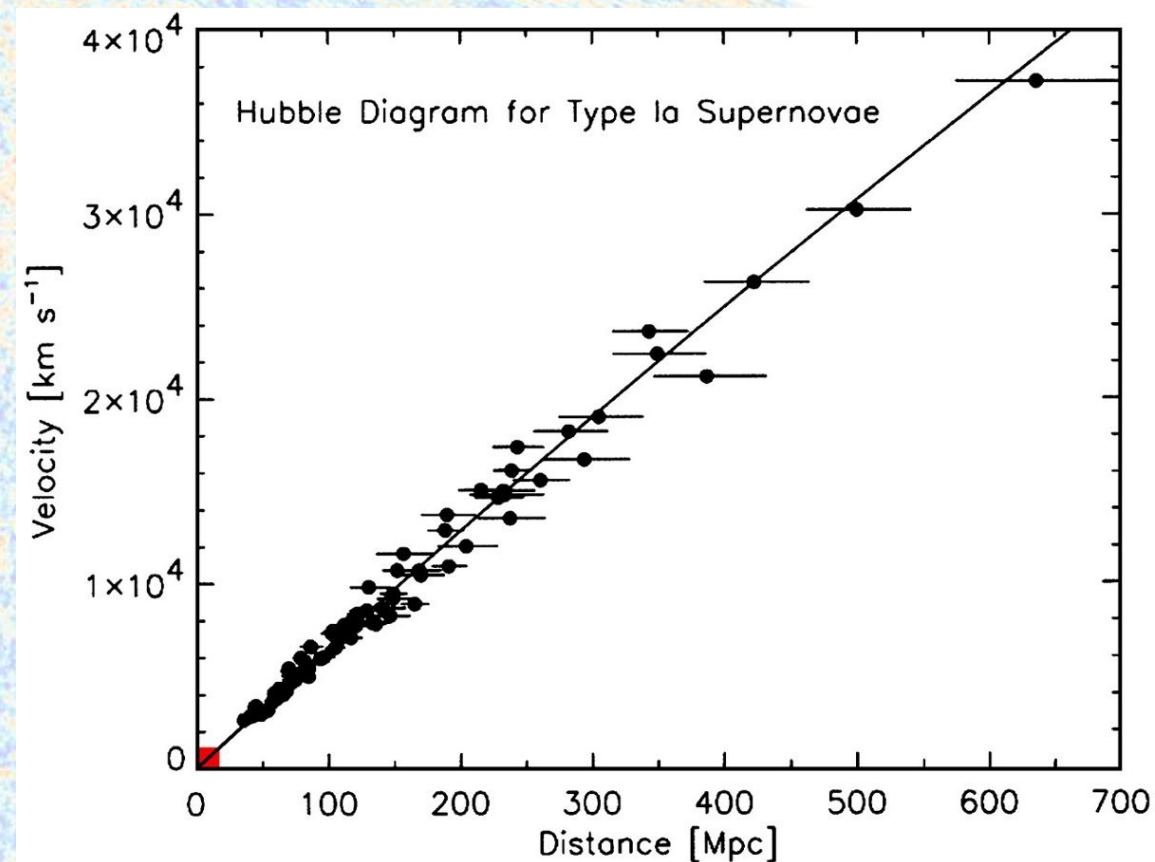


Hubble–Lemaître law and the distance ladder

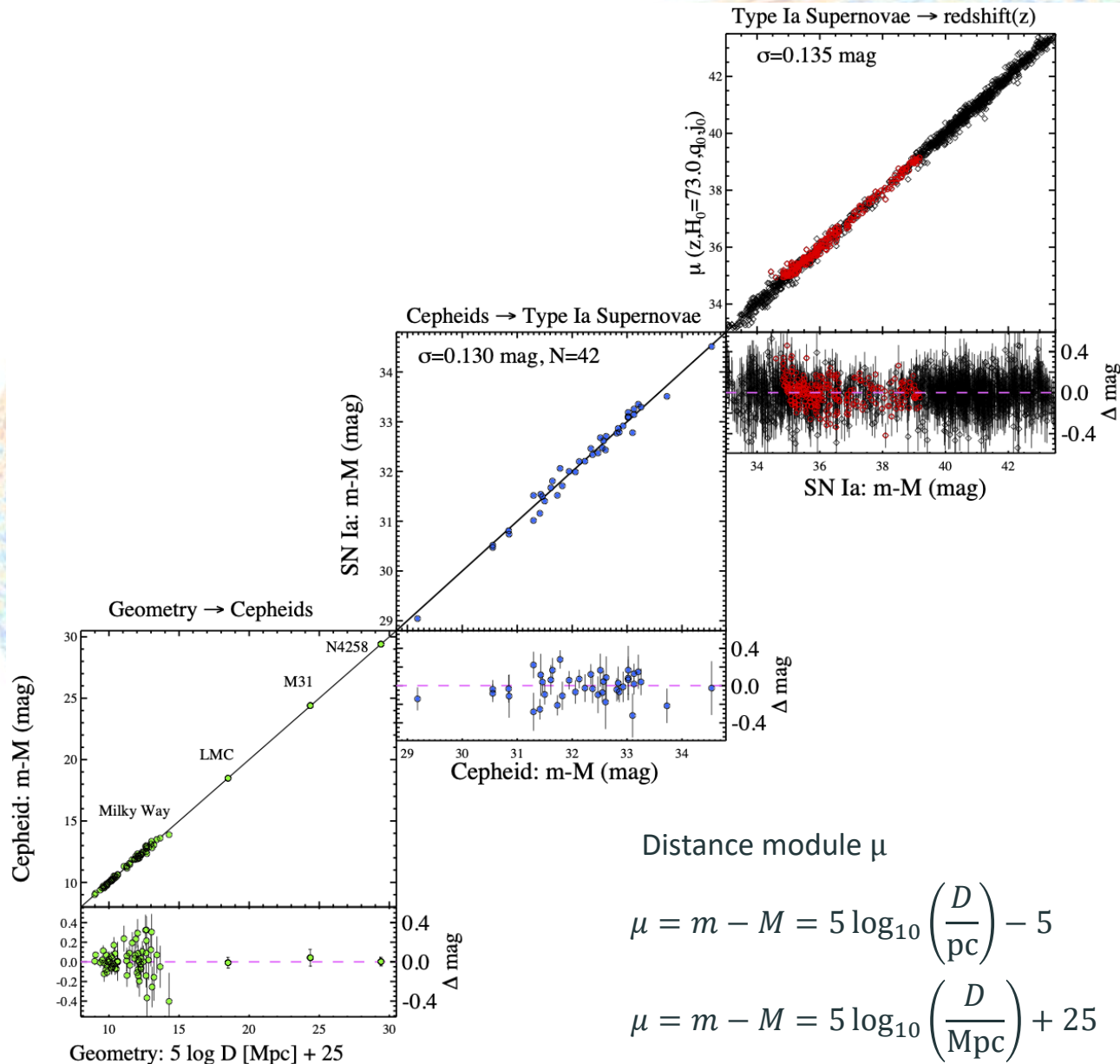
- Velocity: easy, from the spectrum ($v \approx cz$ for $z \ll 1$).
Distance: hard - perhaps astronomy's central problem.
- Ladder: parallax $\rightarrow$ Cepheids (Leavitt 1912: period $\leftrightarrow$ luminosity) $\rightarrow$ SNe Ia.

Careful: Cosmological redshift is **not** an ordinary Doppler shift!

- **Doppler redshift:** Caused by the relative motion of the source and observer through space.
- **Cosmological redshift:** Caused by the expansion of the Universe during the propagation of light.
- As the Universe expands, the wavelength of freely propagating light increases in proportion to the cosmic scale factor: $\lambda \propto a(t)$.
- Consequently, photons observed today have lower frequencies and energies than when they were emitted.
- **Key distinction:** Galaxies do not need to move through space for cosmological redshift to occur; their proper separation can increase because the geometry of space itself evolves.



Cosmic distance ladder



What is it?

- Three rungs of the SH0ES distance ladder; each rung calibrates the next one up.
- Every point is one distance measured two independent ways; the line has slope 1.
- Lower panels: residuals Δ mag scatter around zero, so the methods agree.

Axes, rung by rung

- **Geometry $\rightarrow$ Cepheids:** x = geometric distance (parallax, LMC, NGC 4258) as $5 \log D + 25$; y = Cepheid distance modulus $m - M$.
- **Cepheids $\rightarrow$ SN Ia:** x = Cepheid $m - M$, y = SN Ia $m - M$ in 42 host galaxies ($\sigma = 0.130$ mag).
- **SN Ia $\rightarrow$ redshift:** x = SN Ia $m - M$; $y = \mu$ from redshift z , assuming $H_0 = 73.0$ ($\sigma = 0.135$ mag).

Hubble's law

- Hubble's law: $v = cz = H_0 d$. Far SNe give z and a calibrated distance d .
- The top panel fits the slope of that relation: $H_0 = 73.0 \pm 1.0 \text{ km/s/Mpc}$.
- This is about 5σ above the CMB value (67.4), the "Hubble tension".

Riess, Yuan et al, 2022

Friedmann equations from a Newtonian trick

Take a sphere of radius $R = a(t)x$ and density ρ , and a test galaxy at its edge (mass m):

$$\frac{1}{2}m\dot{R}^2 - \frac{GMm}{R} = E, \quad M = \frac{4\pi}{3}R^3\rho \Rightarrow \boxed{H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}}$$

($E \propto -k$: total energy of the expanding sphere.) General relativity adds that *pressure gravitates too*:

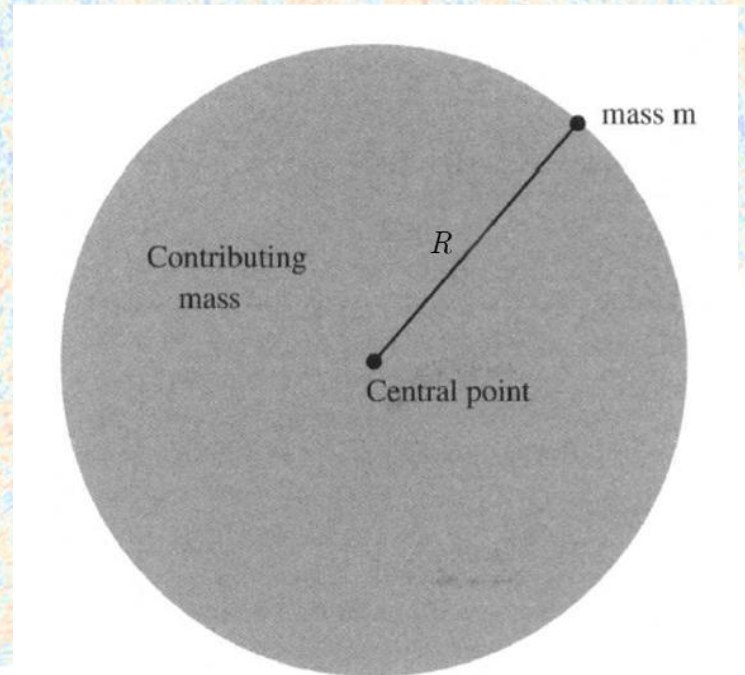
$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)}$$

Critical density: the curvature switch ($k = 0$)

$$\rho_c = \frac{3H_0^2}{8\pi G} \approx 9 \times 10^{-27} \text{ kg m}^{-3} \approx 5 \text{ protons per m}^3. \quad \Omega \equiv \rho/\rho_c:$$

$\Omega > 1$ closed, $\Omega = 1$ flat, $\Omega < 1$ open.

$$\dot{\rho} + 3H\left(\rho + \frac{p}{c^2}\right) = 0$$



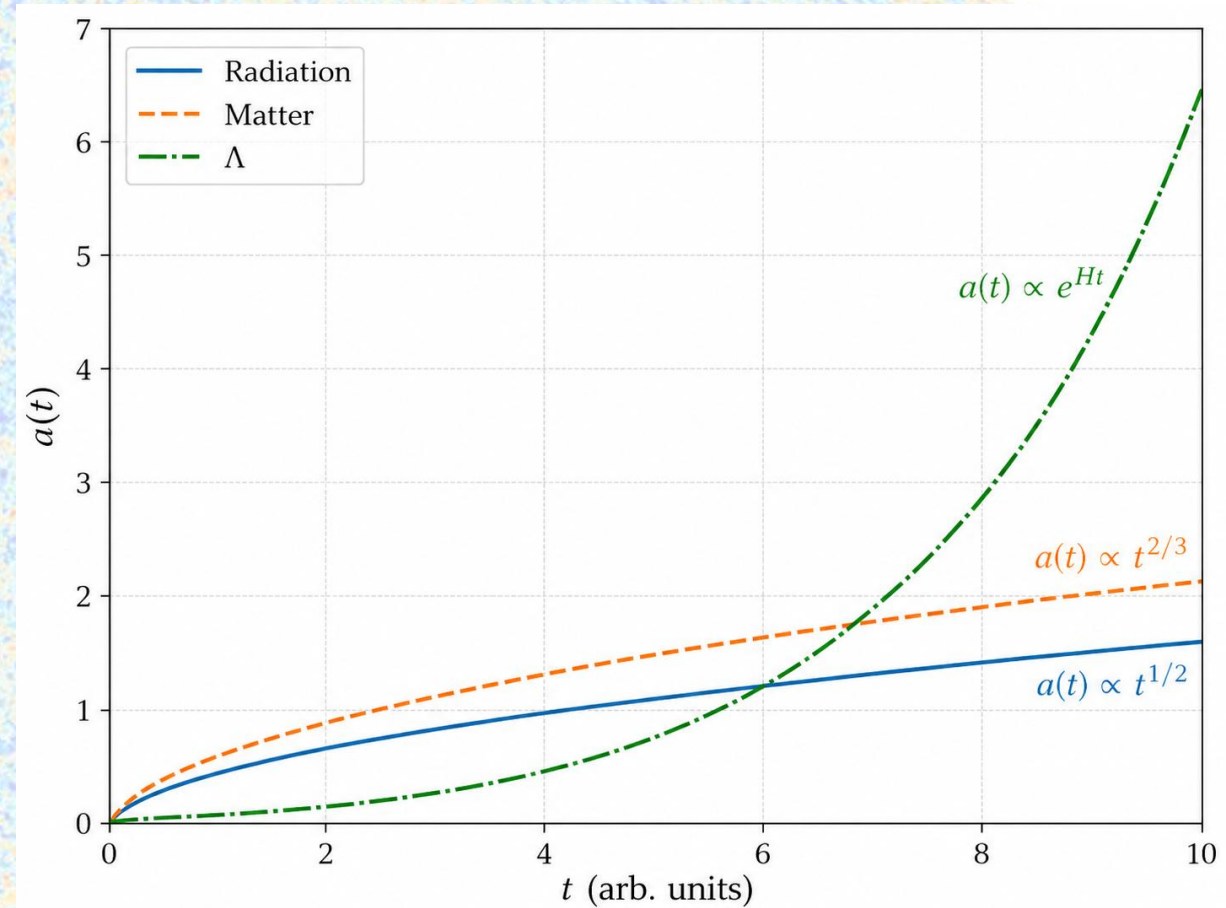
What fills the Universe determines how it expands

Equation of state $p = w\rho c^2$ and energy conservation give $\rho \propto a^{-3(1+w)}$:

	w	$\rho(a)$	$a(t)$
Radiation	$1/3$	a^{-4}	$t^{1/2}$
Matter	0	a^{-3}	$t^{2/3}$
Λ (vacuum)	-1	const	e^{Ht}

Why: photons dilute by volume *and* lose energy by redshift ($a^{-3} \cdot a^{-1}$). Vacuum energy does not dilute at all.

Acceleration needs $\rho + 3p < 0$, i.e. $w < -1/3$.



Age of the Universe: three estimates

All three use $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The answer depends on how the expansion rate changed over time.

1. Hubble time

Assumes the expansion rate never changed: no gravity, no Λ .

$$t_H = \frac{1}{H_0} = \frac{3.09 \times 10^{19} \text{ s}}{70} \\ \approx 4.4 \times 10^{17} \text{ s}$$

$\approx 14.0 \text{ Gyr}$

If expansion slowed down, the true age is below $1/H_0$.

2. Einstein–de Sitter

Flat, matter only ($\Omega_m = 1$). Gravity slows the expansion.

$$a \propto t^{2/3} \Rightarrow H = \frac{2}{3t} \\ t_0 = \frac{2}{3H_0} = \frac{2}{3} \times 14.0 \text{ Gyr}$$

$\approx 9.3 \text{ Gyr}$

Younger than the oldest stars ($\approx 12\text{--}13 \text{ Gyr}$): the “age problem”.

3. Flat Λ CDM

$\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$. Expansion first slows, then accelerates. $t_0 = \int_0^\infty \frac{dz}{(1+z)H(z)}$

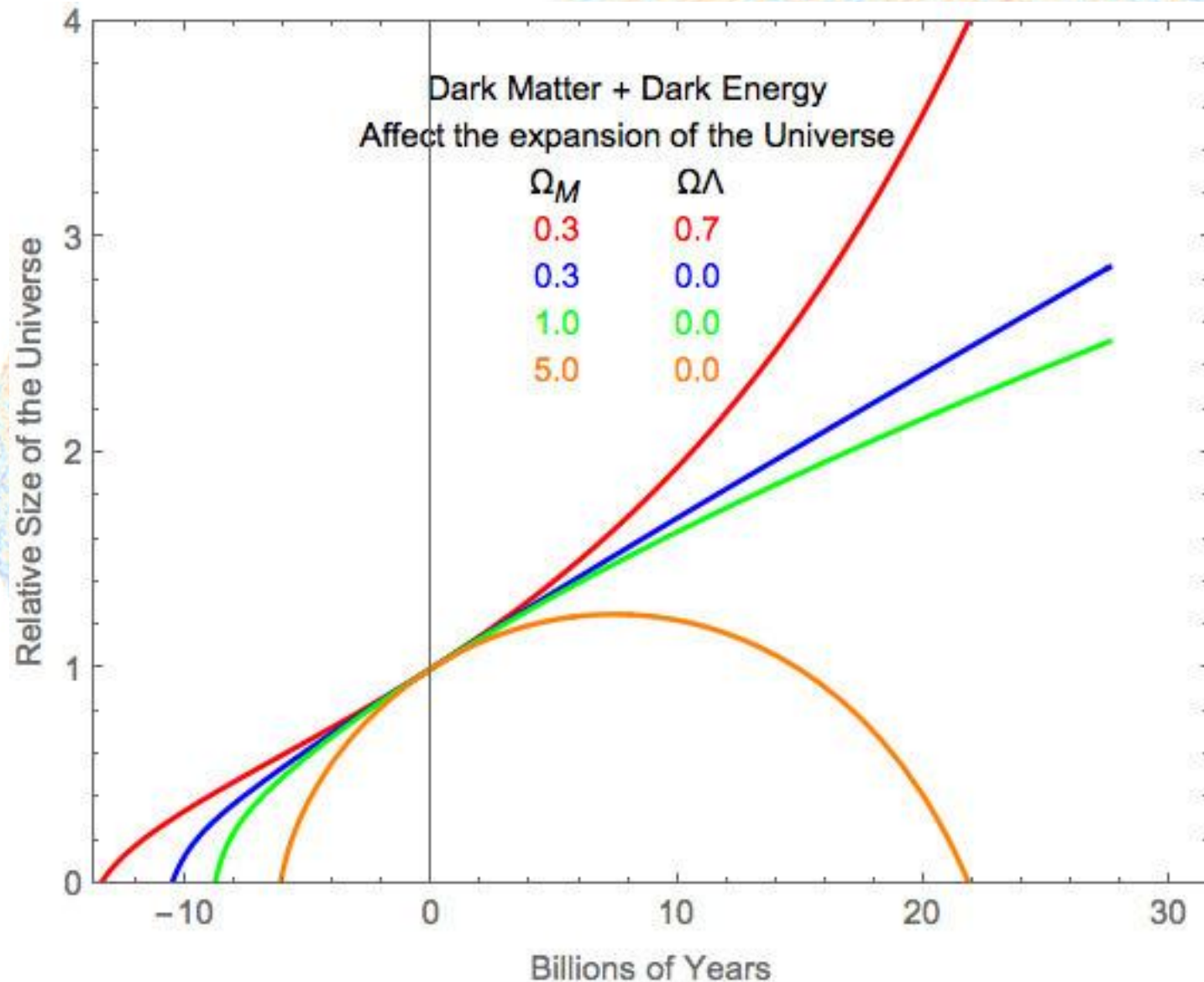
$$t_0 = \int_0^1 \frac{da}{aH(a)} \\ = \frac{2}{3H_0\sqrt{\Omega_\Lambda}} \operatorname{arsinh} \sqrt{\frac{\Omega_\Lambda}{\Omega_m}}$$

$\approx 13.5 \text{ Gyr}$

Planck 2018: 13.8 Gyr ($H_0 = 67.4$, $\Omega_m = 0.315$).

General flat model: $H(a) = H_0 \sqrt{(\Omega_m a^{-3} + \Omega_\Lambda)}$. $1 \text{ Mpc} = 3.09 \times 10^{19} \text{ km}$; $1 \text{ yr} = 3.16 \times 10^7 \text{ s}$.

Friedmann Models



$$\Omega_0 = \Omega_{rel} + \Omega_b + \Omega_d + \Omega_k + \Omega_\Lambda$$
$$\Omega_0 = \rho/\rho_c$$

What the plot shows

- Size of the Universe $a(t)$ vs. time; $a = 1$ today. All curves use $H_0 \approx 70$, so they share today's slope.
- A curve meets $a = 0$ at the Big Bang; that gap is the age.

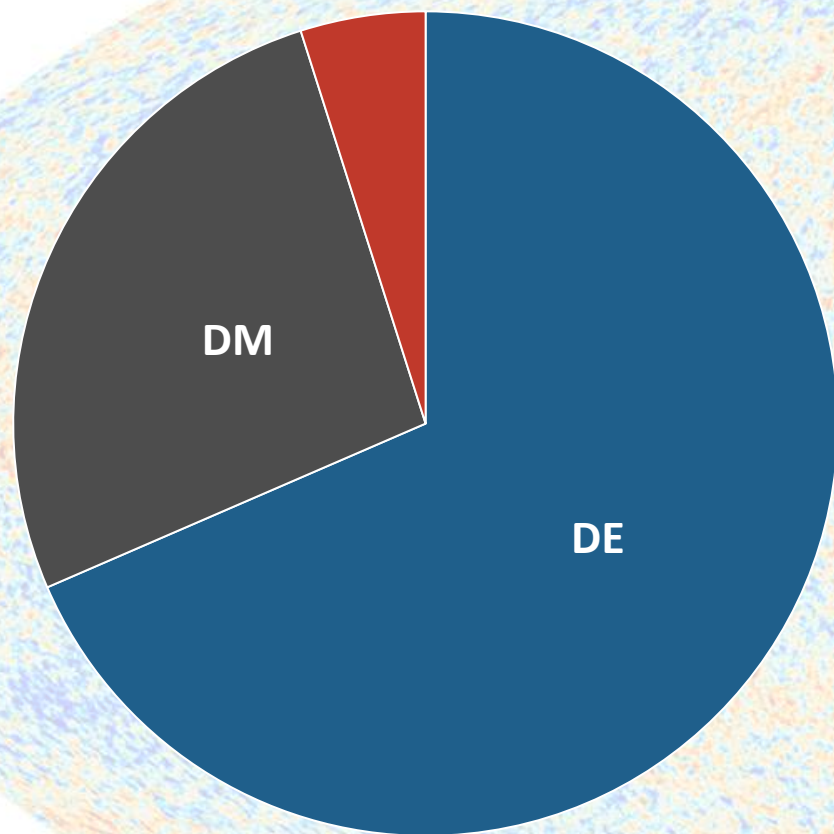
Four Friedmann models

- **Red, Λ CDM:** slows, then accelerates; ≈ 13.5 Gyr.
- **Blue, open:** expands forever, almost linearly; ≈ 11 Gyr.
- **Green, Einstein–de Sitter:** $a \propto t^{2/3}$; 9.3 Gyr.
- **Orange, closed:** recollapses ≈ 22 Gyr from now; ≈ 6 Gyr.

Takeaway

- More matter: stronger deceleration, younger Universe. Λ makes the expansion accelerate.
- $\rho_c = 3H_0^2/8\pi G$; $\Omega_0 = 1$ means a flat Universe.

The Universe's recipe (Planck)



- Dark energy $\Omega_\Lambda = 68.5\%$
- Dark matter $\Omega_c = 26.4\%$
- Ordinary matter $\Omega_b = 4.9\%$ — everything we are made of
- Age $t_0 = 13.8$ Gyr, $\Omega_{\text{tot}} \approx 1$ (flat to $\pm 0.2\%$)

Evidence for dark matter: galaxy rotation curves (Rubin), cluster dynamics (Zwicky), Bullet Cluster. Evidence for Λ : SNe Ia acceleration (1998), BAO, CMB.

Run the film backwards: the hot Big Bang

Expansion $\Rightarrow$ the past was denser *and hotter*: $T \propto 1/a = (1 + z) T_0$.

Epoch	Time	Energy scale
Inflation	$\sim 10^{-34}$ s	$\sim 10^{15}$ GeV
Electroweak symmetry breaking	$\sim 10^{-11}$ s	~ 100 GeV
Quark–hadron transition	$\sim 10^{-5}$ s	~ 150 MeV
Neutrino decoupling	1 s	1 MeV
Nucleosynthesis	3 min	0.1 MeV
Matter-radiation equality	$\sim 5 \times 10^4$ yr	≈ 0.8 eV ($z \approx 3400$)
Recombination, CMB released	380 000 yr	0.3 eV ($z \approx 1100$)
First stars, reionization	10^8 yr	$z \approx 25 \rightarrow 6$
Expansion accelerates / dark energy dominates	$\approx 8 / 10$ Gyr	$z \approx 0.6 / 0.3$
Formation of the Solar System	10 Gyr	0.5
Physics for Physicists (P4P)	14 Gyr	0

Rules of thumb: $1 \text{ eV} \leftrightarrow 11\,600 \text{ K}$. In the radiation era $t \approx 1 \text{ s} \times (1 \text{ MeV}/kT)^2$.

The oldest light: CMB

- Gamow, Alpher, Herman (1948): a relic glow must exist. Dicke, Peebles & co. in Princeton are preparing to look...
- 1964: Penzias and Wilson (Bell Labs) find an unexplained, isotropic noise: ≈ 3.5 K. (They first blame pigeon droppings.)
- Today $T_0 = 2.725$ K: a perfect black body.
- Temperature differences: $\Delta T/T \sim 10^{-5}$ - **seeds of galaxies**.
- COBE (1992), WMAP, Planck (2013–2018).
- Nobel Prizes: 1978 (Penzias, Wilson), 2006 (COBE), 2019 (Peebles: theoretical discoveries in physical cosmology).

Estimate: when was it released?

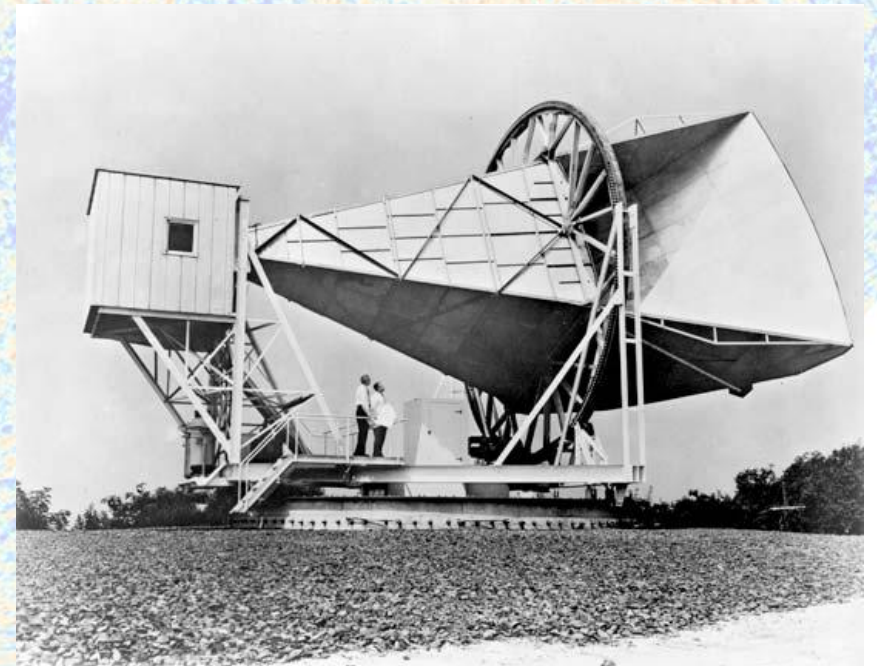
Hydrogen ionizes at 13.6 eV ($\approx 160\,000$ K), but with $\sim 10^9$ photons per atom the energetic tail keeps it ionized down to $T_{\text{rec}} \approx 3000$ K (0.26 eV).

So $1 + z_{\text{rec}} = 3000/2.725 \approx 1100$.

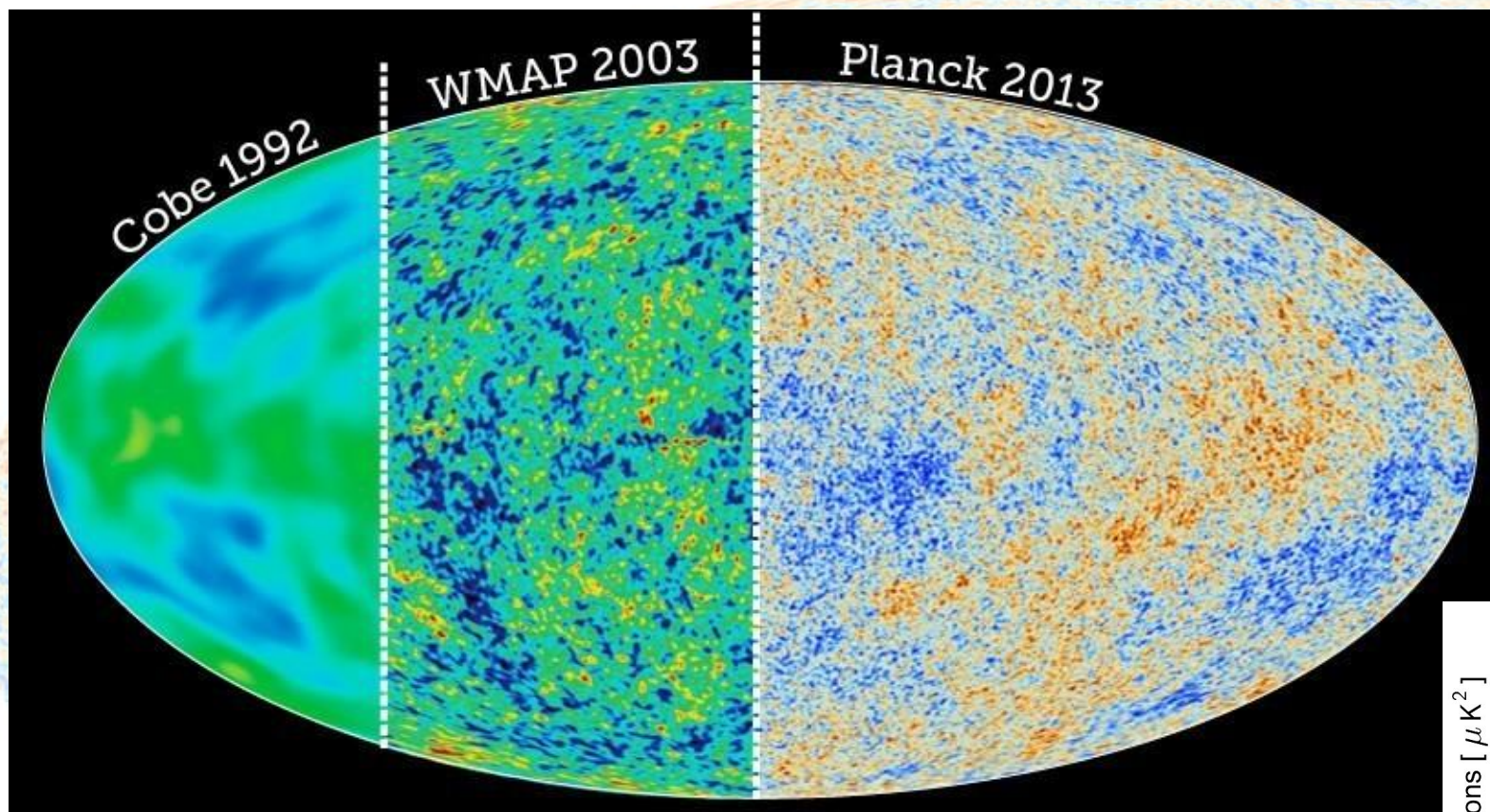
$$a \propto t^{2/3} \Rightarrow t \propto (1 + z)^{-3/2}$$

$$t_{\text{rec}} \approx 13.8 \text{ Gyr} / 1100^{(3/2)} \approx 4 \times 10^5$$

(Λ CDM $\approx 380\,000$ years)

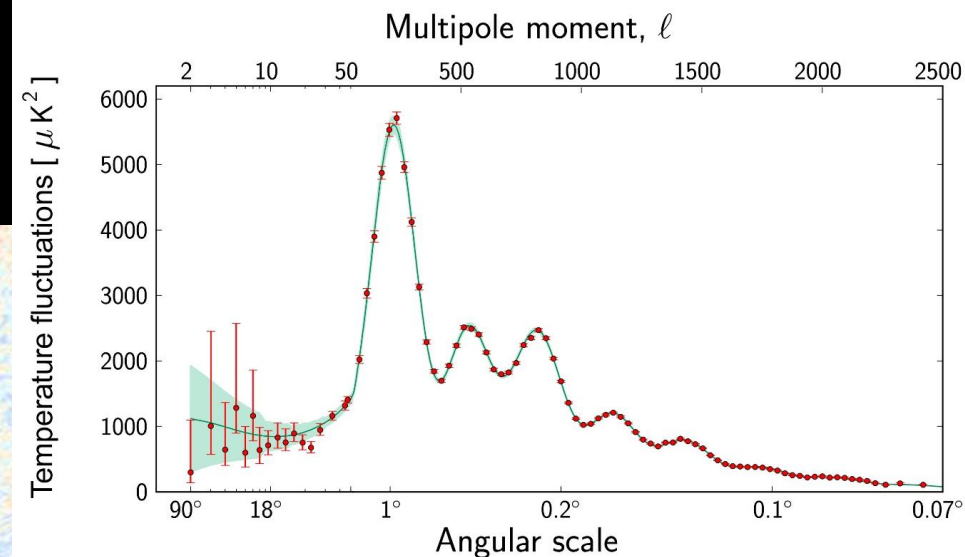
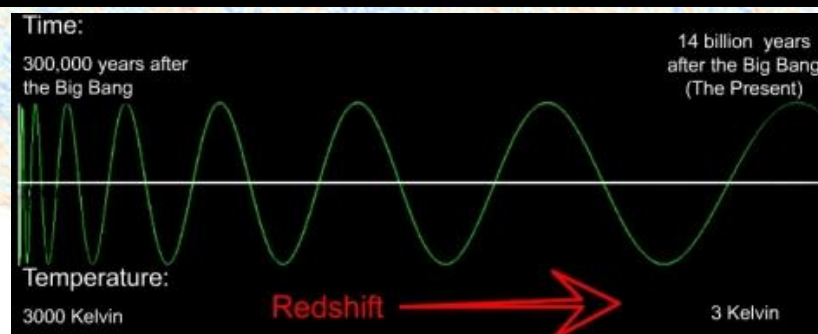


The oldest light: CMB



Why the spectrum has peaks

- Before recombination, photons and baryons were one fluid: gravity compressed it, photon pressure pushed back. Sound waves.
- At 380 000 yr the waves froze. Peaks are scales caught at maximum compression or rarefaction.
- **1st peak, $\ell \approx 220$ ($\sim 1^\circ$):** sound horizon; its position shows space is flat.
- **Higher peaks:** baryon and dark-matter densities.

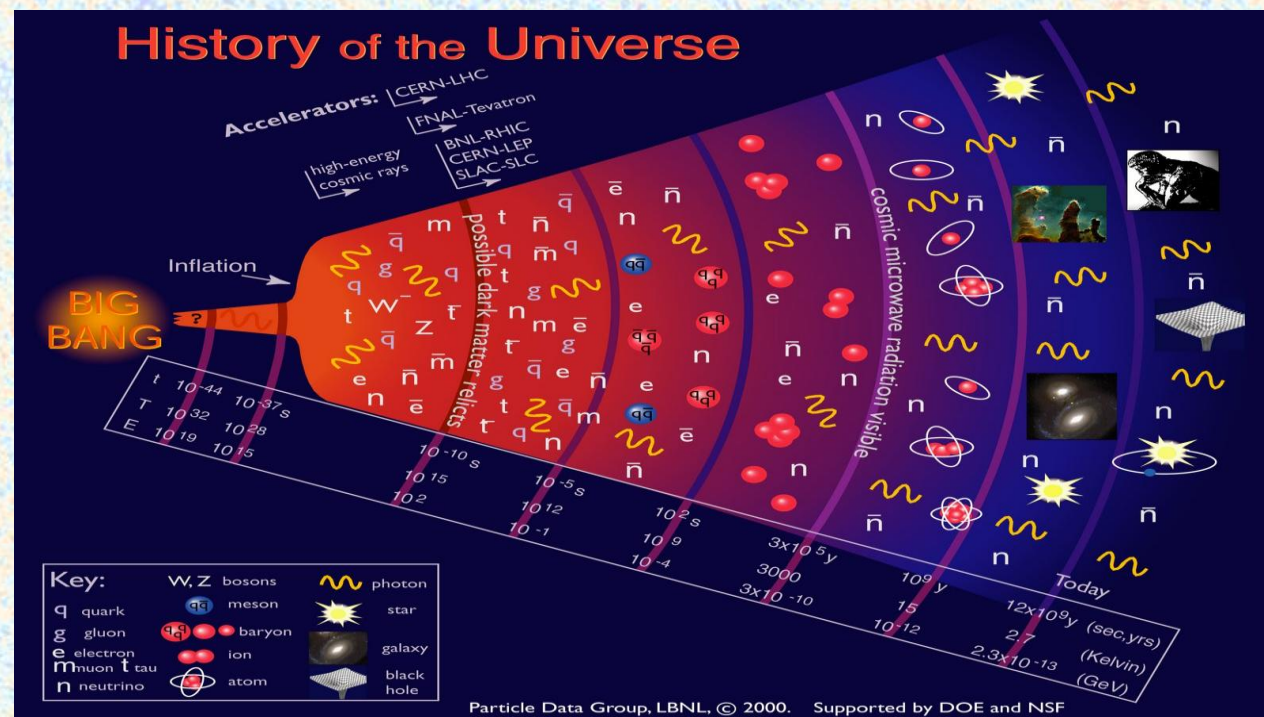


The Standard Model works - but asks uncomfortable questions

Three classic problems of the hot Big Bang, all about **initial conditions**:

1. **Horizon problem.** Why is the CMB the same temperature everywhere?
2. **Flatness problem.** Why is $\Omega \approx 1$?
3. **Origin of perturbations.** Where did the 10^{-5} seeds come from - and why are they nearly scale-invariant?

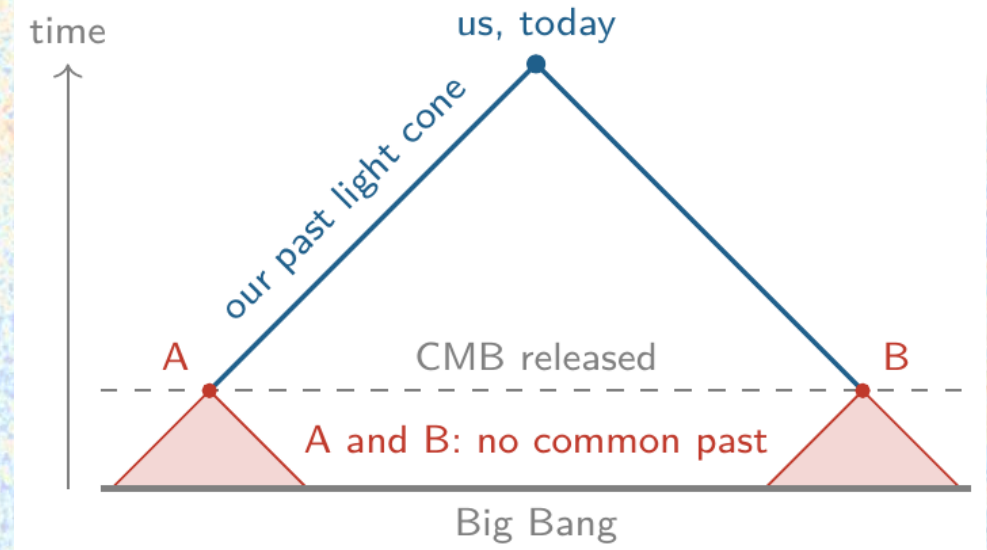
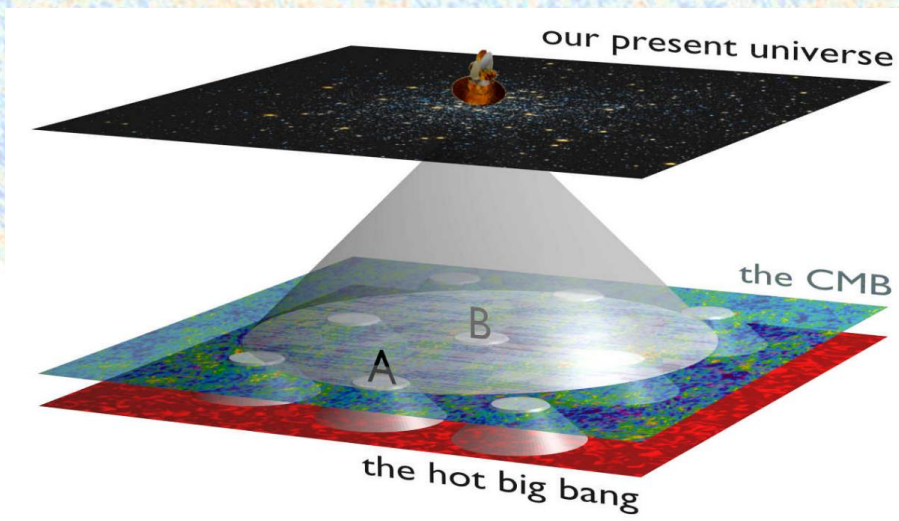
(Plus relics, e.g. magnetic monopoles predicted by grand unified theories.)



1. Horizon problem

- By recombination (380 000 yr) a light signal could cross only a region that is today ≈ 0.3 Gpc across.
- We see that region from ≈ 14 Gpc away: angle $\approx 0.3/14 \approx 0.02$ rad, about 1° .
- The full sky contains $\sim 10^4$ such patches that could *never* exchange a signal — yet they share $T = 2.7255$ K to 1 part in 10^5 .

Thermal equilibrium without communication?



Conformal diagram: light moves at 45° (not to scale).

2. Flatness problem

From the first Friedmann equation: $\Omega - 1 = \frac{kc^2}{a^2 H^2}$.

- Radiation era: $aH \propto 1/a \Rightarrow |\Omega - 1| \propto a^2 \propto t$. Matter era: $|\Omega - 1| \propto a \propto t^{2/3}$.
- Any tiny curvature **grows**: $\Omega = 1$ is an *unstable* fixed point, like a pencil balanced on its tip.

Estimate

Today $|\Omega - 1| \lesssim 0.01$. Running backwards to nucleosynthesis ($t \sim 1$ s): $|\Omega - 1| \lesssim 10^{-16}$. To the Planck time: $\lesssim 10^{-60}$.

The Universe must have started balanced to 1 part in 10^{16} or better - “fine tuning”.

$$\rho_c = \frac{3H^2}{(8\pi G)}, \Omega = \frac{\rho}{\rho_c}$$
$$H^2 = \left(\frac{8\pi G}{3}\right)\rho - \frac{kc^2}{a^2}$$

3. Where do the structures come from?

- CMB: $\Delta T/T \sim 10^{-5}$, with a characteristic pattern - acoustic peaks.
- Sunyaev & Zel'dovich and Peebles & Yu (1970) predicted the pattern: *sound waves* in the hot photon-baryon plasma, frozen at recombination. First peak measured in 2000 (BOOMERanG, MAXIMA); WMAP and Planck mapped it in detail.
- The overdensities grew (with help from dark matter) into galaxies, clusters, and the cosmic web.
- But the standard model has no mechanism to *make* such correlated fluctuations on scales larger than the horizon.

All three problems point to the same thing

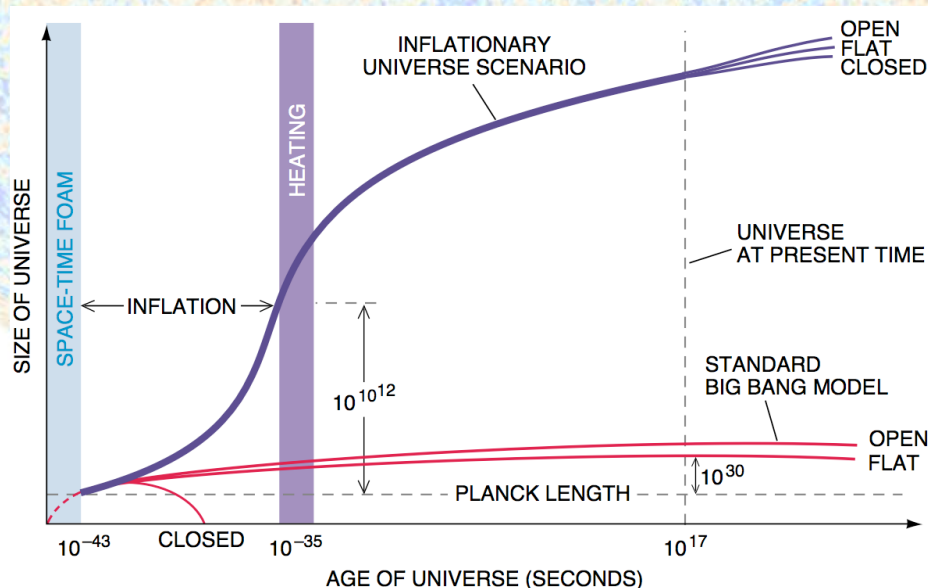
We need a phase *before* the hot Big Bang in which a tiny region became huge, smooth, flat and slightly lumpy.

Cosmic inflation (Starobinsky 1980, Guth 1981, Sato 1981, Linde 1982)

A brief phase of **accelerated** expansion:

$$\ddot{a} > 0 \Leftrightarrow \rho + 3p < 0 \Leftrightarrow \frac{d}{dt} \left(\frac{1}{aH} \right) < 0$$

- Nearly constant H : $a(t) \propto e^{Ht}$.
- Number of e-folds $N = \ln(a_{\text{end}}/a_i) \gtrsim 60$: size grows by $e^{60} \approx 10^{26}$ in $\sim 10^{-34}$ s.



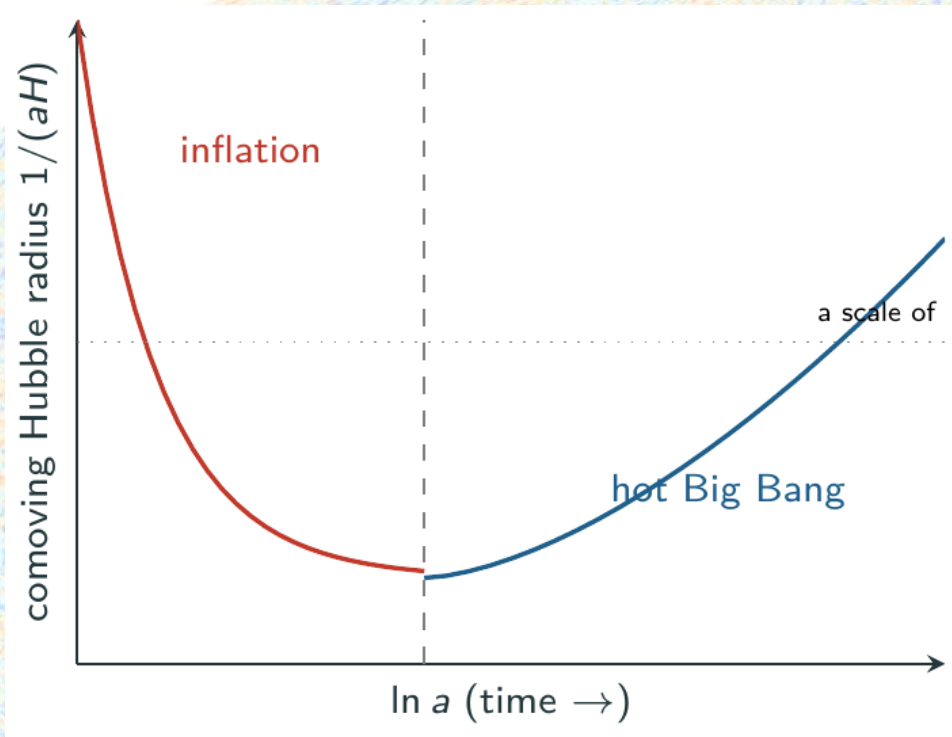
Not the economy's inflation...

Latin *inflare*, “to swell”. (Yugoslavia, Jan. 1994: prices doubled every ≈ 34 h; the Universe doubled every $\sim 10^{-36}$ s, $e^{60} \approx 2^{87}$).

Estimate

A proton (10^{-15} m) $\times 10^{26} \approx 10^{11}$ m
roughly the Earth-Sun distance.

How inflation solves the problems



- **Horizon:** the comoving Hubble radius *shrinks*: a region initially in causal contact is pushed far outside it, then re-enters later.
- **Flatness:** $|\Omega - 1| \propto (aH)^{-2}$ shrinks by $e^{-2N} \sim 10^{-52}$ - curvature erased.
- **Relics:** diluted away.
- **Perturbations:** quantum vacuum fluctuations of the field are stretched to astronomical scales.

What drives inflation? A scalar field

Canonical scalar field ϕ (the *inflaton*) with potential $V(\phi)$:

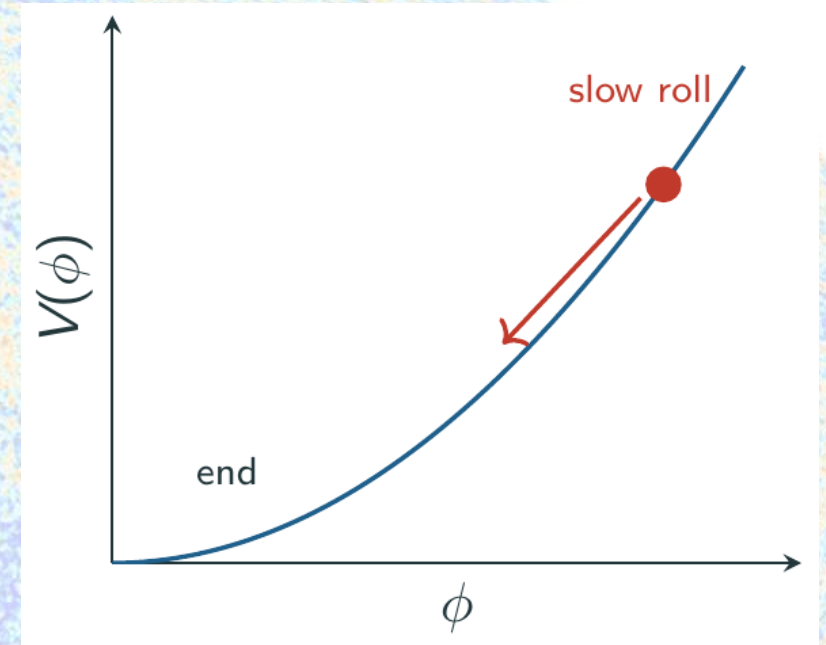
$$\rho = 1/2 \dot{\phi}^2 + V, \quad p = 1/2 \dot{\phi}^2 - V$$

If $\dot{\phi}^2 \ll V$ then $p \approx -\rho$: acts like Λ , so $a \propto e^{Ht}$.

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0 \quad (\text{a ball with friction } 3H)$$

$$H^2 = 8\pi G/3 \rho$$

$$\text{Slow roll: } 3H\dot{\phi} \simeq -V', \quad H^2 \simeq \frac{8\pi G}{3} V.$$



From potential to observables

Hubble hierarchy: $\varepsilon_1 = -\frac{\dot{H}}{H^2}$, $\varepsilon_{i+1} = \frac{d\ln|\varepsilon_i|}{dN}$. Inflation lasts while $\varepsilon_1 < 1$, and ends at $\varepsilon_1 \simeq 1$.

Two numbers measured in the CMB

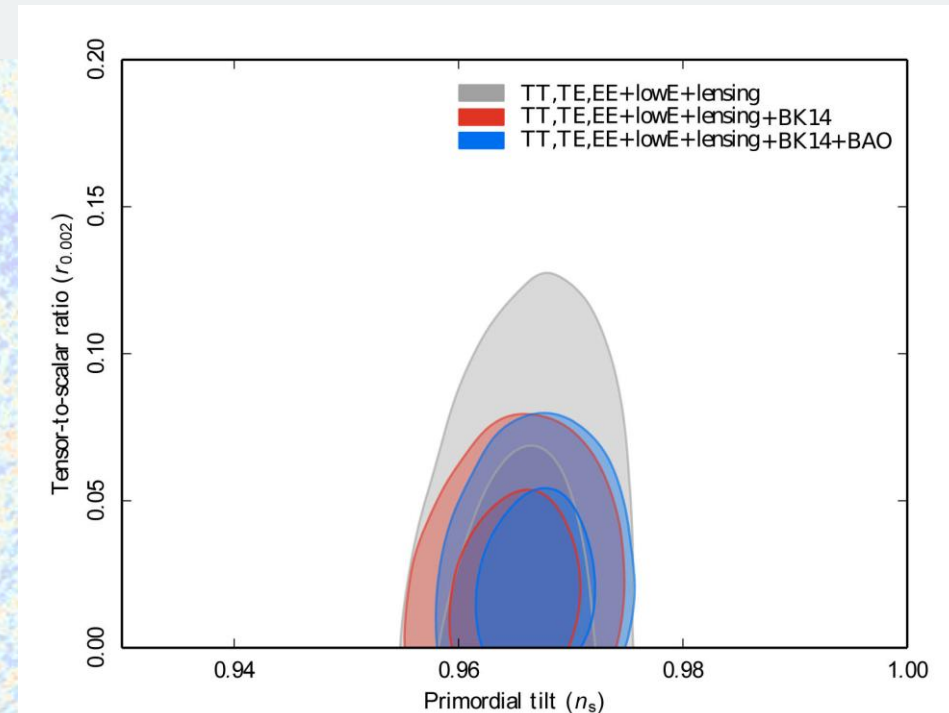
$$r = 16 \varepsilon_1 \quad (\text{tensor-to-scalar ratio}), \quad n_s = 1 - 2\varepsilon_1 - \varepsilon_2 \quad (\text{scalar spectral index})$$

evaluated when the CMB scales left the horizon, $N \approx 50\text{--}60$ e-folds before the end. ($A_s \approx 2.1 \times 10^{-9}$ is the third one.)

Textbook example $V = 1/2 m^2 \phi^2$: $n_s = 1 - 2/N$, $r = 8/N$. For $N = 60$: $n_s = 0.967$, $r = 0.13$.

Starobinsky R^2 : $n_s = 1 - 2/N$, $r = 12/N^2 \approx 0.003$.

Planck 2018 + BICEP/Keck: $n_s = 0.965 \pm 0.004$, $r < 0.036 \Rightarrow 1/2 m^2 \phi^2$ is ruled out, Starobinsky fits.



Planck's verdict and the search for the model

- Simplest single-field models: many are excluded - but **too many others still fit**.
- Theory is built by *postulating* a model and comparing (n_s, r) to the data.
- Energies $\sim 10^{15}$ GeV are far beyond any accelerator: cosmology is the only test.
- New data keep the game moving: BICEP/Keck push r down ($r < 0.036$); ACT DR6 + Planck + DESI (2025) give $n_s = 0.974 \pm 0.003$, about 2σ above Starobinsky's 0.967.
- Next: Simons Observatory (Chile, observing now), LiteBIRD (satellite, 2030s).

The question of our group

Can inflation come from something **other than the canonical scalar field** — from a *tachyon*, a *brane*, or *holography* - and still match Planck?

Our small research group:

Neven Bilić (IRB, Zagreb); Goran Djordjević, Dragoljub Dimitrijević (PMF, Niš); Marko Stojanović (Med. Faculty, Niš).

Lagrangian of a scalar field - $\mathcal{L}(X, \varphi)$

- In general case – any function of a scalar field φ and kinetic energy $X \equiv \frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi$.

- Canonical field with potential $V(\varphi)$

$$\mathcal{L}(X, \varphi) = BX - V(\varphi),$$

- Non-canonical models

$$\mathcal{L}(X, \varphi) = BX^n - V(\varphi),$$

- Dirac-Born-Infeld (DBI) Lagrangian

$$\mathcal{L}(X, \varphi) = -\frac{1}{f(\varphi)} \sqrt{1 - 2f(\varphi)X} - V(\varphi),$$

- Special case – tachyonic $\mathcal{L}(X, \varphi) = -V(\varphi)\sqrt{1 - 2\lambda X}$,

Mechanics: $L = T - V = \frac{1}{2} m \dot{q}^2 - V(q) \rightarrow$ field: $q \rightarrow \varphi$, $\frac{1}{2} m \dot{q}^2 \rightarrow X = \frac{1}{2} \dot{\varphi}^2$

Relativistic particle: $L = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} \rightarrow$ same square-root form as DBI.

Tachyon inflation

- Classic “tachyon”: hypothetical particle faster than light. Modern meaning (A. Sen, string theory): a field with negative m^2 that rolls toward a minimum (*tachyon condensation*) - instability, not superluminal signals.
- Dirac–Born–Infeld Lagrangian and, with $\dot{\theta}^2 < 1$,

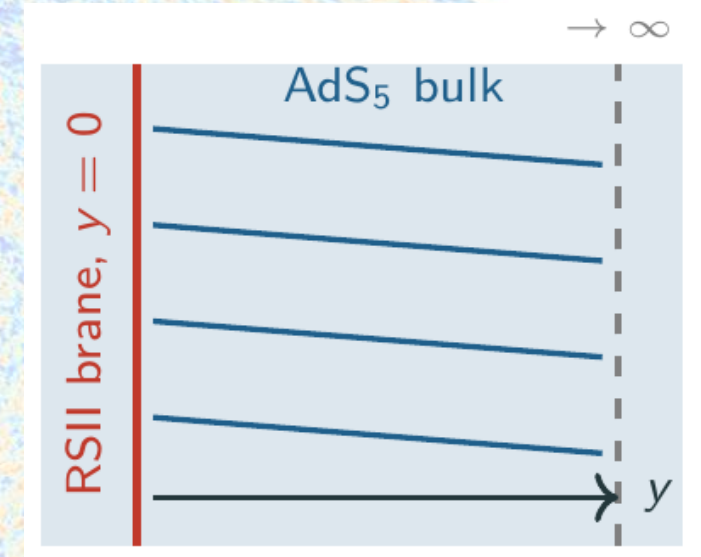
$$\mathcal{L} = -V(\theta)\sqrt{1 - \dot{\theta}^2}, \quad p = -V\sqrt{1 - \dot{\theta}^2}, \quad \rho = \frac{V}{\sqrt{1 - \dot{\theta}^2}}$$

- Equation of state $w = p/\rho = \dot{\theta}^2 - 1 \in [-1, 0]$: from dark-energy-like ($\dot{\theta} \rightarrow 0$) to dust.
- Potential: $V(0) = \text{const}$, $V \rightarrow 0$ as $\theta \rightarrow \infty$: the field “rolls” toward $\theta \rightarrow \infty$.
- One line of algebra (standard FRW):

$$\varepsilon_1 = -\dot{H}/H^2 = 3/2 \dot{\theta}^2. \text{ Inflation } (\varepsilon_1 < 1) \Leftrightarrow \dot{\theta}^2 < 2/3 \Leftrightarrow w < -1/3.$$

Randall–Sundrum braneworld

- Matter lives on a 3-brane in a 5D anti-de Sitter (AdS_5) bulk; only gravity propagates into the bulk.
- RSI (1999): two branes, explains why gravity is so weak. RSII: observer on the positive-tension brane, the other brane pushed to infinity.
- Friedmann on the brane: $H^2 = \frac{8\pi G}{3} \rho \left(1 + \frac{\rho}{2\sigma} \right)$, σ = brane tension. Extra friction at high $\rho \Rightarrow$ longer inflation.
- A dynamical D3-brane moving in the bulk acts as a tachyon with potential $V \propto \theta^{-4}$.
- *Extended RSII*: matter in the bulk (back-reaction) modifies the warp factor $\Rightarrow$ modified cosmology *and* potential.



Holographic braneworld

- AdS/CFT: the brane sits at the *holographic boundary* of asymptotic AdS_5 ; effective 4D theory = boundary CFT + tachyon matter (Bilić).
- Flat FRW, dimensionless $h \equiv \ell H$ (ℓ = AdS radius), $\kappa^2 = 8\pi G_N/\ell^2$. Friedmann equations:

$$h^2 - \frac{h^4}{4} = \frac{\kappa^2}{3} \ell^4 \rho, \quad \dot{h} \left(1 - \frac{h^2}{2} \right) = -\frac{\kappa^2}{2} \ell^3 (p + \rho) \quad (\text{standard: } h^2 = \kappa^2/3 \ell^4 \rho)$$

- Solving for h : $h^2 = 2 \left(1 - \sqrt{1 - \kappa^2 \ell^4 \rho/3} \right)$. The “−” branch recovers standard cosmology when $\kappa^2 \ell^4 \rho \ll 1$.
- The h^4 term encodes the 5D bulk (via the CFT); negligible at low density.

Bonus

$h \leq h_{\text{max}} = \sqrt{2}$, $\rho \leq \rho_{\text{max}} = 3/(\kappa^2 \ell^4)$:
finite density and expansion rate at $t = 0$
- **no Big Bang singularity.**

Numerical machinery: a fast test of models

Hamilton's equations ($\eta = \dot{\theta}/\sqrt{1 - \dot{\theta}^2}$: momentum):

$$\dot{\theta} = \frac{\eta}{\sqrt{1 + \eta^2}}, \quad \dot{\eta} = -3H\eta - \sqrt{1 + \eta^2} \frac{V_{,\theta}}{V}$$

(friction $3H\eta$ + force $V_{,\theta}/V$; the model enters via $H(\rho)$)

1. Draw $\sim 10^4$ random parameter sets: $N \in [60, 90]$, $\omega \in (0, 0.25)$, $\theta_i \in (0, 20)$, κ, h_i (initial conditions: $\eta_i = 0$ or $\dot{\eta}_i = 0$, slow roll).
2. Integrate; compute $\varepsilon_1, \varepsilon_2, \dots$; find t_{end} from $\varepsilon_1 = 1$; step back N e-folds to t_{CMB} .
3. Get (n_s, r) from $\varepsilon_i(t_{\text{CMB}})$ — and, to gain speed, from analytic approximations.

Potentials tested

$$V(\theta) = \frac{1}{\cosh(\omega\theta)},$$

$$V(\theta) = (1 + \theta) e^{-\omega\theta}$$

Models: FLRW tachyon, RSII (extended), holographic.

Result of the study: the approximate formulas reproduce the numerical (n_s, r) very fast, which makes scanning many models cheap.

Three models, one family of potentials

Model	Field and energy density	Expansion rate	Free parameters
Tachyon tachyon field, standard cosmology	$V = \text{sech}(w\theta)$ $\rho = \sqrt{V^2 + \pi_\theta^2}$	$H^2 = \frac{\kappa^2 \rho}{3}$	N_*, κ, w
Braneworld DBI-type field on a Randall–Sundrum II brane	$\chi = \cosh^{1/4} \varphi$ $\rho = \chi^{-4} \sqrt{1 + \chi^8 \pi_\varphi^2}$	$H^2 = \frac{\kappa^2 \rho}{3} \left(\chi' + \frac{\kappa^2 \rho}{12} \right)$	N_*, κ
Holographic tachyon field, holographic cosmology	$V = \text{sech}(w\theta)$ $\rho = V \sqrt{1 + \eta^2}$	$H^2 = 2 - 2 \sqrt{1 - \frac{\kappa^2 \rho}{3}}$	N_*, κ, w, h_i

κ : coupling (gravitational/brane scale); w : width of the potential; h_i : initial Hubble parameter; N_* : e -folds between horizon exit and the end of inflation. Same type of matter, **different cosmology**: the Friedmann equation is what changes.

From a model to (n_s, r) : slow-roll principle

Hubble-flow parameters measure how slowly the expansion changes:

$$\varepsilon_1 = -\frac{\dot{H}}{H^2}, \quad \varepsilon_{i+1} = \frac{\dot{\varepsilon}_i}{H\varepsilon_i}$$

Inflation: $\varepsilon_1 < 1$; slow roll: $|\varepsilon_i| \ll 1$.

All quantities are evaluated at horizon exit, $N_\star$ e -folds before the end of inflation.

Leading order (canonical field):

$$n_s \simeq 1 - 2\varepsilon_1 - \varepsilon_2, \quad r \simeq 16\varepsilon_1$$

Non-canonical fields (tachyon, DBI): perturbations propagate with a **sound speed** $c_s \leq 1$.

$$\mathcal{P}_s \simeq \frac{H^2}{8\pi^2 \varepsilon_1 c_s}, \quad \mathcal{P}_t \simeq \frac{2H^2}{\pi^2}$$
$$r \simeq 16\varepsilon_1 c_s$$

The chain of computation

model $\rightarrow$ background $H(t) \rightarrow \varepsilon_i, c_s$ at $N_\star \rightarrow (n_s, r)$

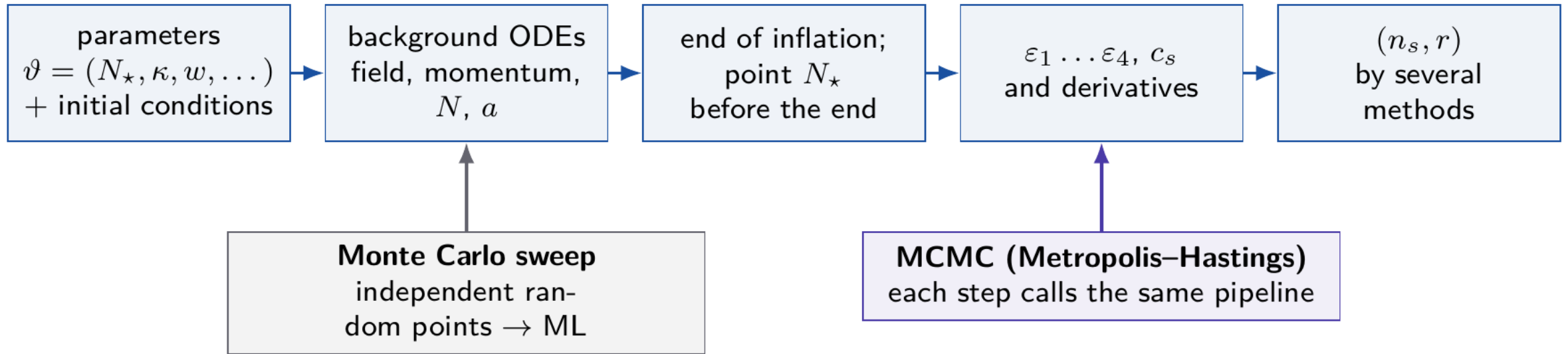
Eight ways to compute (n_s, r) from the same background

Method	Idea	uses c_s
LO	leading-order slow roll: $n_s = 1 - 2\varepsilon_1 - \varepsilon_2$, $r = 16\varepsilon_1$	—
NLO	second-order slow-roll corrections, model-specific coefficients	—
Resummed	sums all powers of ε_1 in the leading terms	—
DBI	leading order with sound-speed flow, $r = 16\varepsilon_1 c_s$	✓
PS	leading amplitudes $\mathcal{P}_s, \mathcal{P}_t$; n_s from their scale dependence, used in MCMC	✓
k-inf. 2nd / 3rd	high-order slow-variation expansion of k -inflation (Zhu et al. 2014)	✓
MS	numerical evolution of the Mukhanov–Sasaki mode equation	✓

$$\text{MS: } v_q'' + \left(c_s^2 q^2 - \frac{z''}{z} \right) v_q = 0, \quad z = \frac{a\sqrt{2\varepsilon_1}}{c_s}, \quad \mathcal{P}_s = \frac{q^3 |v_q|^2}{2\pi^2 z^2}$$

PS is algebraic and fast, so it is the natural choice inside a chain of 10^4 – 10^5 evaluations..

One numerical pipeline for every model



inflationSim: C++20, Boost.odeint (Runge–Kutta–Fehlberg 7(8)), OpenMP.

A new model only supplies its equations, H and V .

One evaluation = one full integration: Holographic ≈ 0.07 – 0.2 s, Tachyon ≈ 0.13 s, Braneworld ≈ 0.7 – 0.9 s.

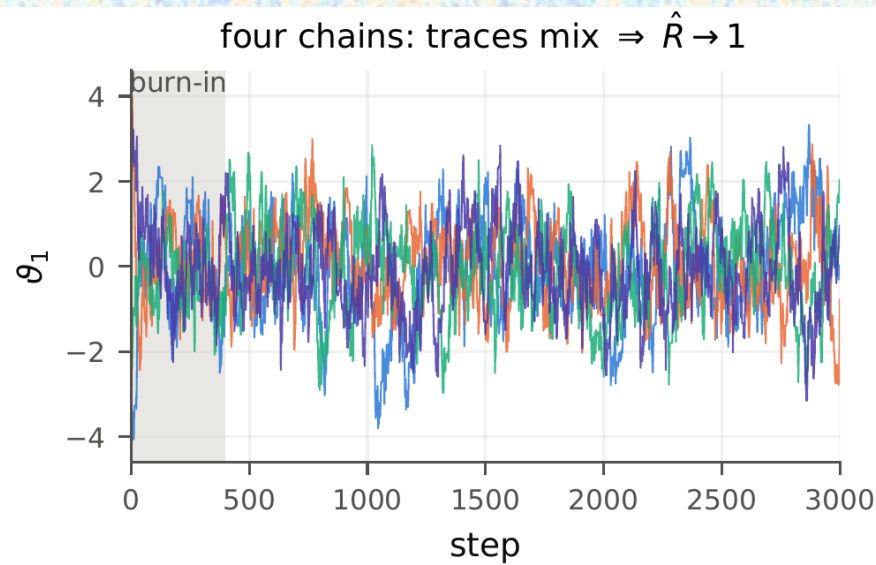
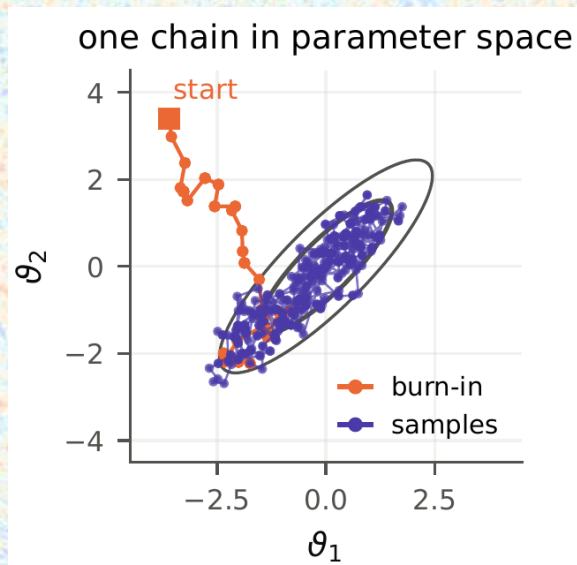
Failed trajectories are rejected: 0.3–1.7 failed integrations per MCMC step.

Markov chain Monte Carlo: the Metropolis–Hastings algorithm

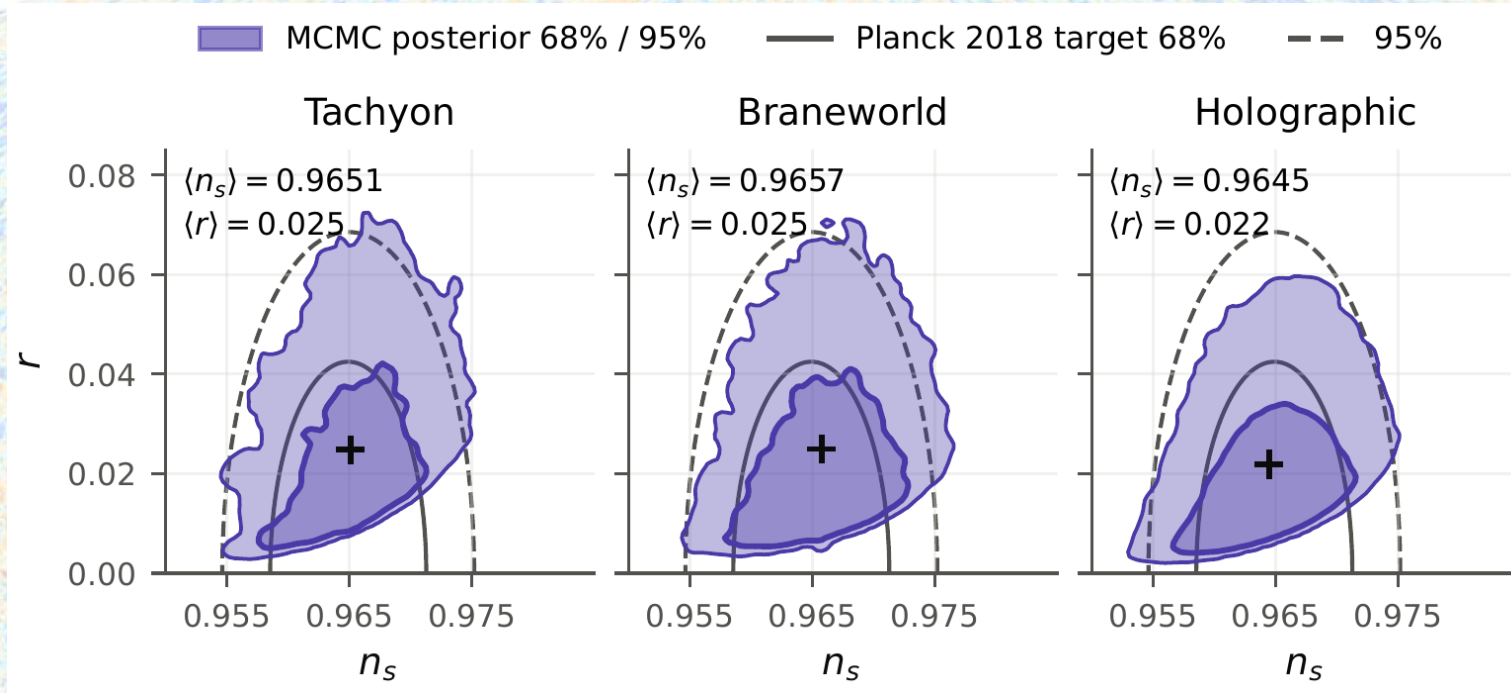
1. Propose a step: $\vartheta' = \vartheta + \delta$, $\delta \sim \mathcal{N}(0, \text{diag } \sigma^2)$.
2. Accept with probability $A = \min(1, p(\vartheta' | D)/p(\vartheta | D))$.
3. If rejected, **repeat** the current state.

Better points are always accepted, worse ones sometimes.

After a **burn-in**, the visited states are distributed as the posterior.



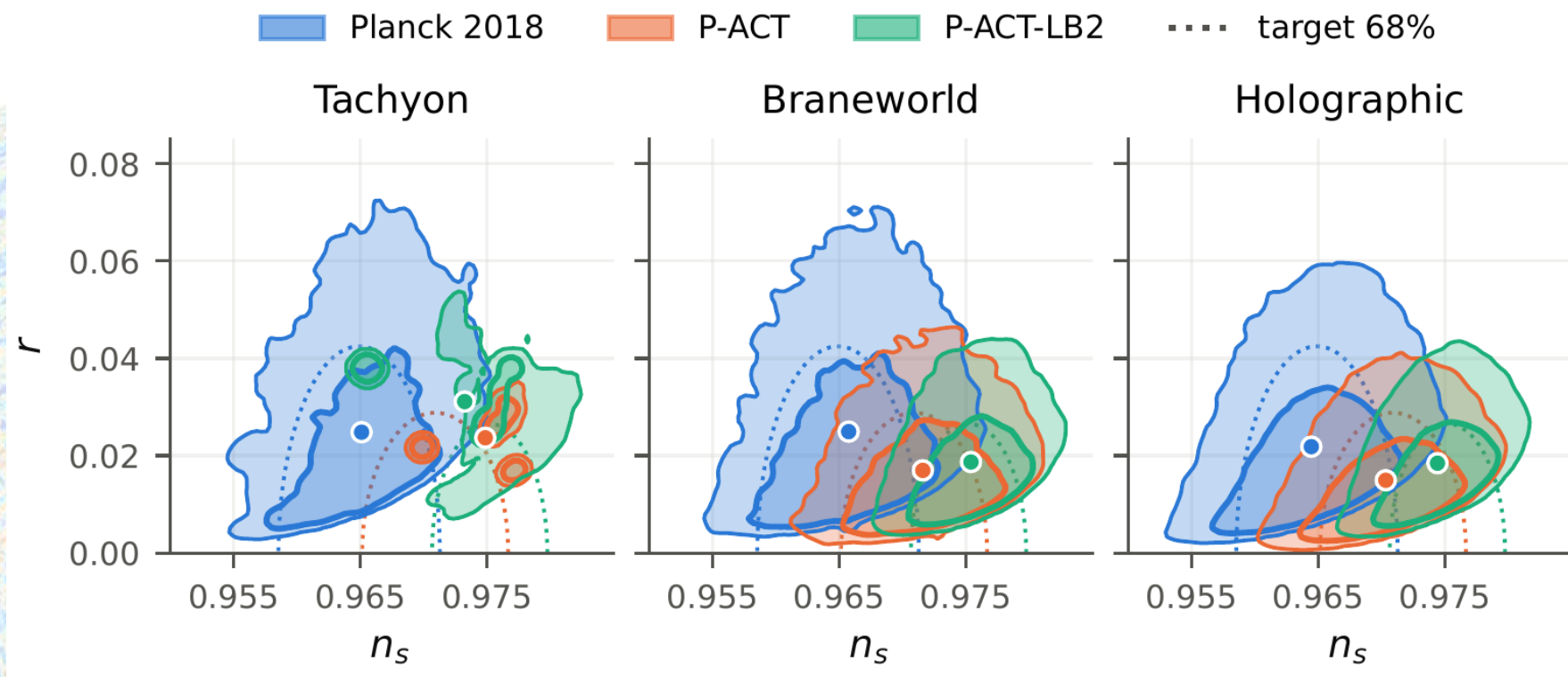
Posteriors in the (n_s, r) plane, Planck 2018



- All three posteriors are centred in the Planck 68% contour: $\langle n_s \rangle \approx 0.965$.
- Posterior mean $\langle r \rangle = 0.022\text{--}0.025$ (ν_{10} : 0.023–0.032), broad spread (SD 0.013–0.017); below the bound.

Pooled post-burn-in states of 4 chains, n_s and r from the PS approximation; +: posterior mean.

From Planck to ACT DR6 and DESI



$\langle n_s \rangle / \langle r \rangle$	Planck 2018	P-ACT	P-ACT-LB2
Tachyon	0.9651 / 0.025	0.9749 / 0.024	0.9732 / 0.031
Braneworld	0.9657 / 0.025	0.9716 / 0.017	0.9754 / 0.019
Holographic	0.9645 / 0.022	0.9703 / 0.015	0.9744 / 0.018

The posteriors move toward higher n_s without a conflict in r . $\langle r \rangle = 0.015\text{--}0.019$ for ACT at $N_\star \leq 150$, but $0.023\text{--}0.028$ at $N_\star \leq 120$.

Take-aways from our results

- Tachyon inflation in a holographic braneworld **fits Planck** (n_s, r) well, especially for large N (60–90).
- Two simple potentials already give points inside the Planck contours.
- The holographic model behaves best among the ones tested; **no initial singularity** as a by-product.
- Fast numerical/approximate estimates let us scan models quickly.
- Next: analytic study of these potentials; better approximations; confront the new ACT/BICEP-Keck data.

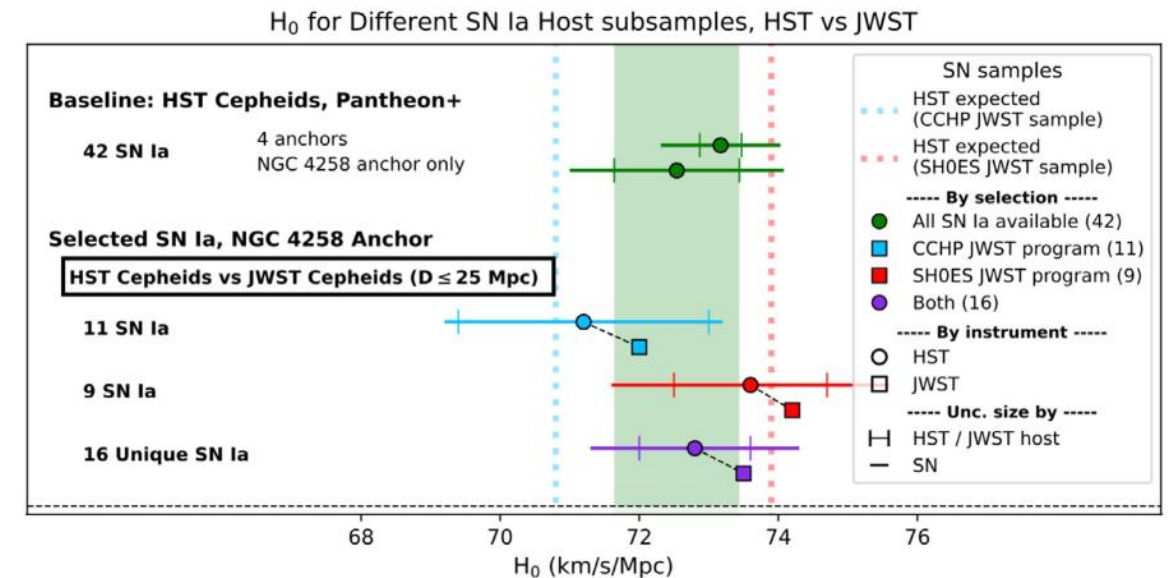
Collaborators: N. Bilić (Zagreb), G. S. Djordjević, D. D. Dimitrijević, M. Stojanović (Niš).

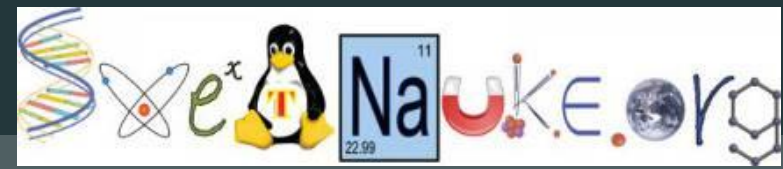
Where we are today: the Hubble tension

- Local ladder (Cepheids + SNe Ia): $H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$.
- CMB (Planck, Λ CDM): $H_0 = 67.36 \pm 0.54 \text{ km s}^{-1} \text{ Mpc}^{-1}$.
- **Difference $\sim 5\sigma$!!!**
- Possible fixes: unknown systematics; *early dark energy*; modified gravity; new physics. (CosmoVerse white paper: 263 pages, ~ 540 authors.)
- DESI (2025): hints ($\approx 3\text{--}4\sigma$) that dark energy is *not* constant.

Lesson

Ptolemy $\rightarrow$ Friedmann $\rightarrow$ inflation: every stage of cosmology answered old questions and opened new ones. The next answer may come from DESI, Euclid, JWST, Rubin (LSST), Simons Observatory, LiteBIRD ...





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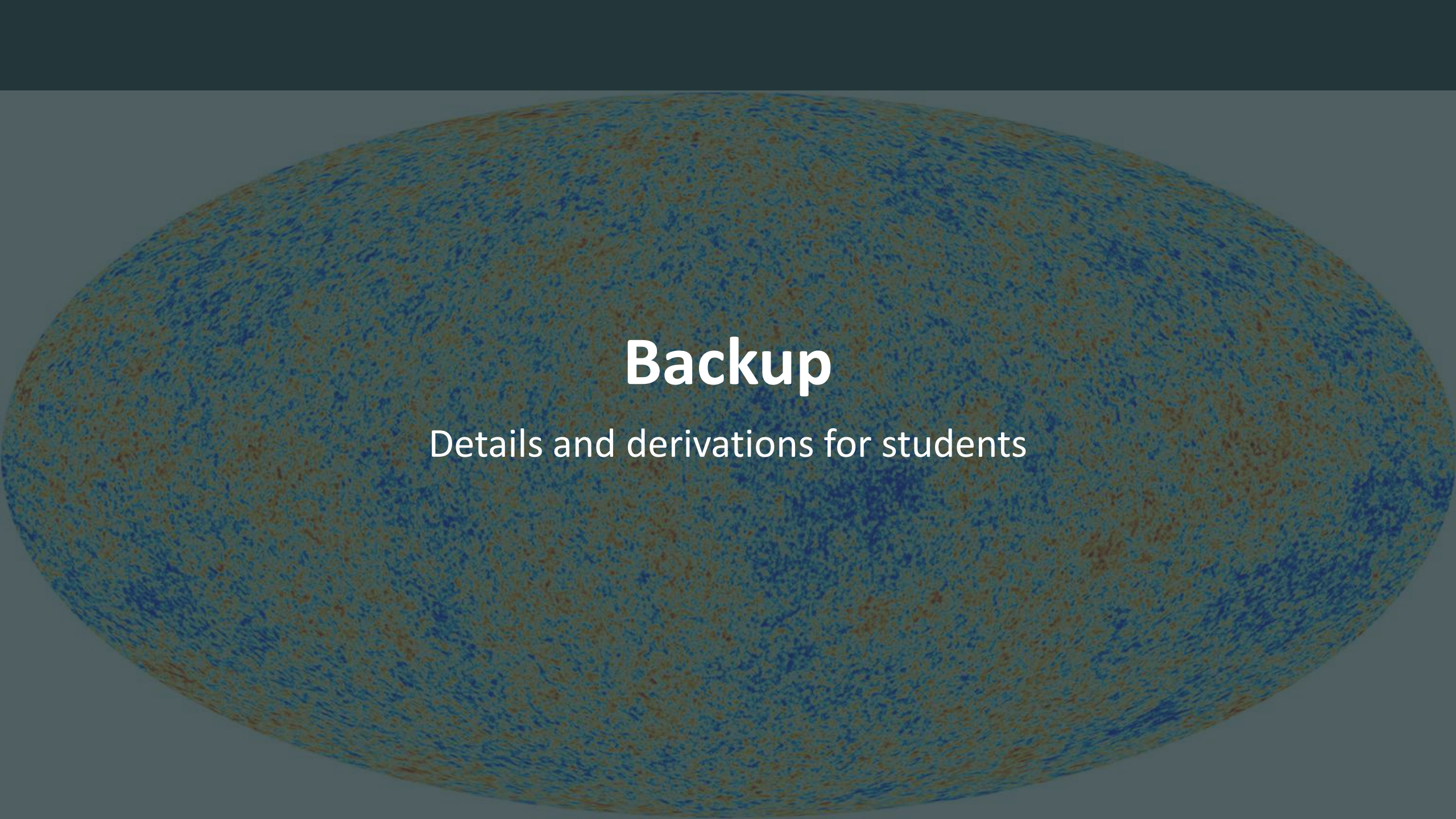
<https://www.facebook.com/mmilann>

Thank you!

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Sources and further reading

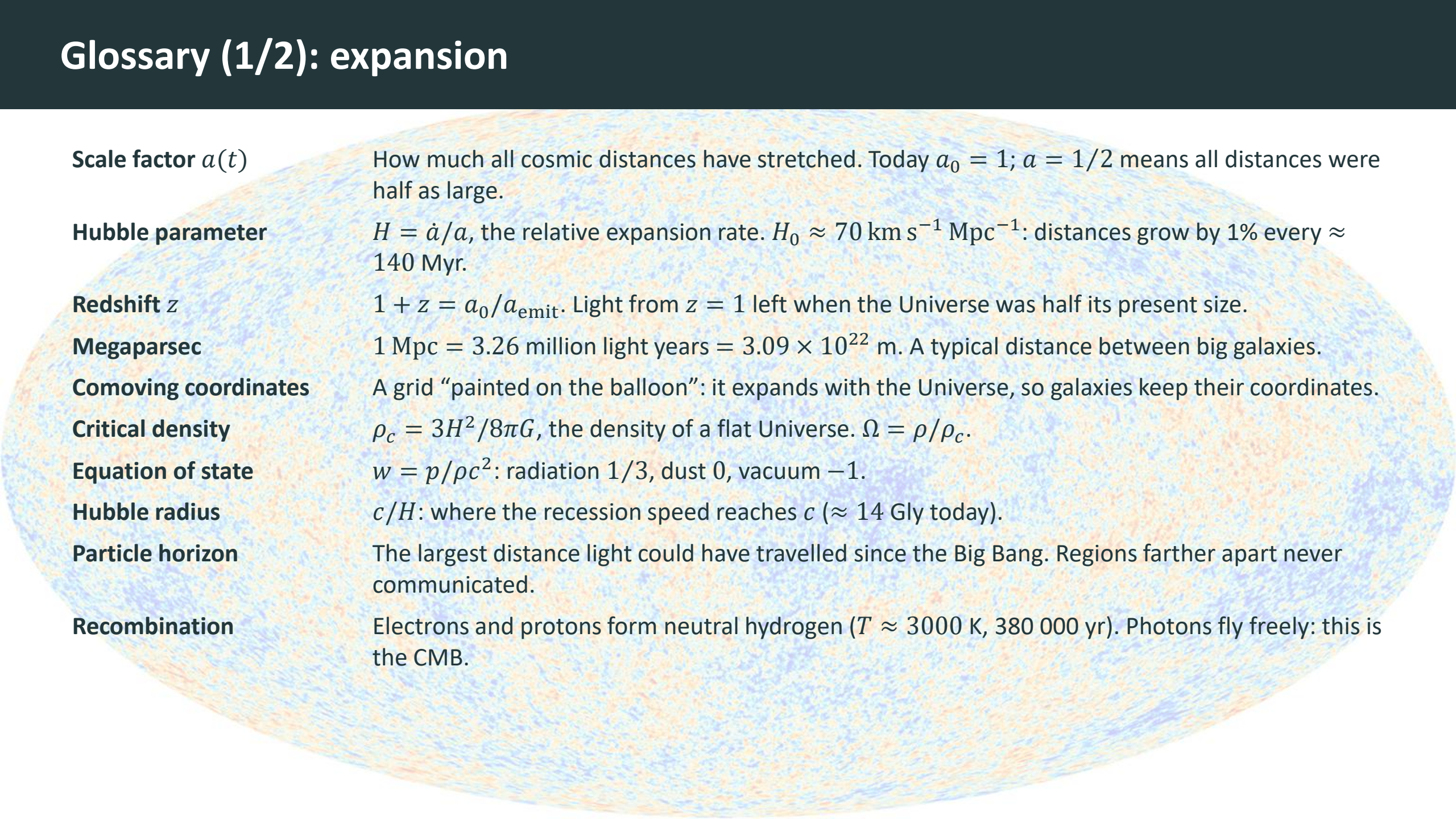
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Backup

Details and derivations for students

Glossary (1/2): expansion



Scale factor $a(t)$	How much all cosmic distances have stretched. Today $a_0 = 1$; $a = 1/2$ means all distances were half as large.
Hubble parameter	$H = \dot{a}/a$, the relative expansion rate. $H_0 \approx 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$: distances grow by 1% every $\approx 140 \text{ Myr}$.
Redshift z	$1 + z = a_0/a_{\text{emit}}$. Light from $z = 1$ left when the Universe was half its present size.
Megaparsec	1 Mpc = 3.26 million light years = $3.09 \times 10^{22} \text{ m}$. A typical distance between big galaxies.
Comoving coordinates	A grid “painted on the balloon”: it expands with the Universe, so galaxies keep their coordinates.
Critical density	$\rho_c = 3H^2/8\pi G$, the density of a flat Universe. $\Omega = \rho/\rho_c$.
Equation of state	$w = p/\rho c^2$: radiation 1/3, dust 0, vacuum -1 .
Hubble radius	c/H : where the recession speed reaches c ($\approx 14 \text{ Gly}$ today).
Particle horizon	The largest distance light could have travelled since the Big Bang. Regions farther apart never communicated.
Recombination	Electrons and protons form neutral hydrogen ($T \approx 3000 \text{ K}$, 380 000 yr). Photons fly freely: this is the CMB.

Glossary (2/2): inflation and beyond

e-fold	Growth by a factor $e \approx 2.718$. N e-folds: factor e^N . 60 e-folds = 10^{26} .
Inflaton	The scalar field whose potential energy drives inflation.
Slow roll	The field moves slowly compared with the expansion, so potential energy dominates and $p \approx -\rho$.
A_s, n_s, r	Amplitude, tilt, and tensor (gravitational-wave) fraction of the primordial perturbations.
Tachyon (modern)	A field with negative mass squared: an <i>instability</i> that rolls to a new vacuum. No faster-than-light signals.
Brane	A (3+1)-dimensional “membrane” (our world) inside a higher-dimensional space, the <i>bulk</i> .
AdS	Anti-de Sitter space: constant negative curvature, radius ℓ .
AdS/CFT	Holography (Maldacena 1997): gravity in 5D AdS is equivalent to a quantum field theory without gravity on its 4D boundary.
$n\sigma$	Statistical significance. 5σ : a fluke with probability ≈ 1 in 3.5 million.

Useful constants for estimates

Distances and times

- $1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$; $1 \text{ pc} = 3.26 \text{ ly} = 3.09 \times 10^{16} \text{ m}$
- $1 \text{ Mpc} = 3.09 \times 10^{22} \text{ m} = 3.09 \times 10^{19} \text{ km}$
- $1 \text{ yr} \approx 3.16 \times 10^7 \text{ s} \approx \pi \times 10^7 \text{ s}$
- $1/H_0 \approx 14 \text{ Gyr}$, $c/H_0 \approx 4.3 \text{ Gpc}$ ($H_0 = 70$)

Energies

- $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J} \leftrightarrow kT$ at 11 600 K
- $m_p c^2 = 938 \text{ MeV}$, $m_e c^2 = 0.511 \text{ MeV}$, $(m_n - m_p)c^2 = 1.29 \text{ MeV}$
- LHC: $\sim 10^4 \text{ GeV}$. Inflation: up to $\sim 10^{16} \text{ GeV}$.

Natural units $\hbar = c = k_B = 1$

- Everything in GeV: $1 \text{ GeV}^{-1} = 6.58 \times 10^{-25} \text{ s} = 1.97 \times 10^{-16} \text{ m}$
- Planck mass $M_{\text{Pl}} = \sqrt{\hbar c/G} = 1.22 \times 10^{19} \text{ GeV}$; reduced $M_{\text{Pl}}/\sqrt{8\pi} = 2.44 \times 10^{18} \text{ GeV}$
- Planck time $5.4 \times 10^{-44} \text{ s}$

The Universe today

- $\rho_c \approx 9 \times 10^{-27} \text{ kg m}^{-3}$ ($\approx 5 \text{ protons/m}^3$)
- CMB: $T_0 = 2.7255 \text{ K}$, 411 photons/cm^3
- Baryons per photon $\eta \approx 6 \times 10^{-10}$

Hubble's law and redshift follow from $a(t)$

Hubble's law in one line. Physical distance $r = a(t) x$ with fixed comoving x :

$$v = \dot{r} = \dot{a} x = \frac{\dot{a}}{a} r = H r$$

Every observer sees the same law: there is no centre.

Redshift. Wavelengths stretch with space, $\lambda \propto a$:

$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} = \frac{a_0}{a_{\text{emit}}}, \quad z \approx \frac{H_0 D}{c} \quad (z \ll 1)$$

Estimate

A galaxy at $D = 100$ Mpc: $v = 70 \times 100 = 7000 \text{ km s}^{-1}$, $z \approx 0.023$.

Faster than light?

$v = Hr > c$ for $r > c/H_0 \approx 14$ Gly. No contradiction: nothing moves *through* space faster than c ; space itself stretches. Galaxies with $z \gtrsim 1.5$ recede faster than c today, and we still see them.

From energy conservation to $a(t)$

1. Fluid equation. First law for a comoving volume $V \propto a^3$: $d(\rho c^2 V) = -p dV$, so

$$\dot{\rho} + 3H \left(\rho + \frac{p}{c^2} \right) = 0 \quad \xrightarrow{p=w\rho c^2} \quad \frac{d\rho}{\rho} = -3(1+w) \frac{da}{a} \Rightarrow \rho \propto a^{-3(1+w)}$$

2. Solve Friedmann (flat): $\dot{a}^2 \propto a^2 \rho \propto a^{-1-3w}$, so

$$a(t) \propto t^{2/[3(1+w)]}; \quad w = 1/3 \rightarrow t^{1/2}, \quad w = 0 \rightarrow t^{2/3}, \quad w \rightarrow -1 \rightarrow e^{Ht}$$

3. Acceleration equation: differentiate Friedmann and use step 1 $\Rightarrow \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + \frac{3p}{c^2} \right)$.

Estimate: the age problem

With $H_0^{-1} = 14.5$ Gyr (Planck), a matter-only Universe has $t_0 = 2/3 H_0^{-1} \approx 9.7$ Gyr, younger than the oldest stars (≈ 12 – 13 Gyr). With Λ : $t_0 \approx 0.95 H_0^{-1} \approx 13.8$ Gyr. Another hint for dark energy.

Olbers: how far would we need to see?

A line of sight hits a stellar disc after a *mean free path*

$$\lambda = \frac{1}{n \pi R_{\odot}^2}$$

- $n \approx 10^9$ stars per Mpc^3 (10^{11} stars per galaxy, $\sim 10^{-2}$ galaxies per Mpc^3) $\approx 3 \times 10^{-59} \text{ m}^{-3}$
- $\pi R_{\odot}^2 \approx 1.5 \times 10^{18} \text{ m}^2$
- $\lambda \approx 2 \times 10^{40} \text{ m} \approx 2 \times 10^{24} \text{ ly}$

We see out to only $\sim 5 \times 10^{10} \text{ ly}$: stellar discs cover a fraction $\sim 10^{-14}$ of the sky. Stars also live only $\sim 10^{10} \text{ yr}$.

The twist

In every direction our line of sight *does* end on a glowing surface: the CMB plasma at 3000 K, as hot as a cool star. Olbers was right, but the expansion has redshifted that glow by a factor 1100, down to 2.7 K. It is invisible to the eye.

Why does hydrogen recombine at 3000 K, not 160 000 K?

- Binding energy $B = 13.6$ eV, but there are $\eta^{-1} \approx 1.6 \times 10^9$ photons per baryon.
- A fraction $\sim e^{-B/kT}$ of photons can ionize. Hydrogen stays ionized while there is at least ~ 1 ionizing photon per atom.

Saha equation (ionized fraction x_e , baryon density $n_b = \eta n_\gamma$):

$$\frac{x_e^2}{1 - x_e} = \frac{1}{n_b} \left(\frac{m_e kT}{2\pi\hbar^2} \right)^{3/2} e^{-B/kT}$$

Half-ionized when the right-hand side is ≈ 1 :

$$\frac{B}{kT} \approx \underbrace{\ln \frac{1}{0.24 \eta}}_{\approx 23} + \underbrace{3/2 \ln \frac{m_e c^2}{2\pi kT}}_{\approx 19} \approx 41 \Rightarrow kT \approx 0.33 \text{ eV} \approx 3800 \text{ K}$$

Photons decouple a little later, at $T \approx 3000$ K ($z \approx 1100$, $t \approx 380\,000$ yr).

Horizon problem and the magic number 60

The angle. Comoving distances (“today’s size”):

- horizon at recombination: $\int_0^{t_{\text{rec}}} c \, dt/a \approx 280 \text{ Mpc}$
- distance to the CMB: $\approx 14\,000 \text{ Mpc}$
- angle $\theta \approx 280/14\,000 \approx 0.02 \text{ rad} \approx 1.2^\circ$
- number of causal patches $\sim 4\pi/\theta^2 \sim 10^4$

Why 60 e-folds? Today’s Hubble radius must fit inside one causal patch at the start of inflation: $e^N \gtrsim a_0 H_0 / (a_e H_e)$.

- radiation era: $aH \propto a^{-1}$; matter era: $aH \propto a^{-1/2}$
- $a_e H_e / (a_0 H_0) \approx \sqrt{a_{\text{eq}}} / a_e$, with $a_{\text{eq}} \approx 3 \times 10^{-4}$ and $a_e \approx T_0 / T_e \approx 2 \times 10^{-28}$ for $T_e \sim 10^{15} \text{ GeV}$
- ratio $\approx 10^{26} = e^{60} \Rightarrow N \gtrsim 60$

Slow roll: from the potential to (n_s, r)

Potential slow-roll parameters ($M_{\text{Pl}} = 2.44 \times 10^{18}$ GeV, reduced):

$$\epsilon_V = \frac{M_{\text{Pl}}^2}{2} \left(\frac{V'}{V} \right)^2, \quad \eta_V = M_{\text{Pl}}^2 \frac{V''}{V}$$

To first order $\epsilon_1 \approx \epsilon_V$, $\epsilon_2 \approx 4\epsilon_V - 2\eta_V$, so

$$n_s = 1 - 6\epsilon_V + 2\eta_V, \quad r = 16\epsilon_V$$

$$N = \int H dt \approx \frac{1}{M_{\text{Pl}}^2} \int_{\phi_{\text{end}}}^{\phi} \frac{V}{V'} d\phi$$

Worked example: $V = 1/2 m^2 \phi^2$

$\epsilon_V = \eta_V = 2M_{\text{Pl}}^2/\phi^2$; inflation ends at $\phi_{\text{end}}^2 = 2M_{\text{Pl}}^2$.

$N = \phi^2/4M_{\text{Pl}}^2 - 1/2 \Rightarrow \epsilon_V = \eta_V \approx 1/2N \Rightarrow n_s = 1 - 2/N, r = 8/N$.

Field range: $\phi_{\text{CMB}} \approx \sqrt{4N} M_{\text{Pl}} \approx 15 M_{\text{Pl}}$ — a “large-field” model, hence its large r .

$A_s = 2.1 \times 10^{-9}$ fixes $m \approx 6 \times 10^{-6} M_{\text{Pl}} \approx 1.5 \times 10^{13}$ GeV.

From quantum jitter to galaxies

- During inflation the field fluctuates quantum-mechanically, $\delta\phi \sim H/2\pi$, on all scales.
- Each mode is stretched beyond the Hubble radius, *freezes*, and re-enters much later as a density perturbation.
- Scalar spectrum $P_s(k) = A_s (k/k_*)^{n_s-1}$. $n_s = 1$ (Harrison–Zel’dovich) means the same on all scales. Inflation predicts n_s slightly below 1, because H slowly decreases: *inflation has to end*.
- Tensor modes are primordial gravitational waves, $P_t = r P_s$. They would leave a **B-mode** (curl-like) polarization pattern in the CMB.

Estimate: r measures the energy of inflation

$$V^{1/4} \approx 1.0 \times 10^{16} \text{ GeV} \times \left(\frac{r}{0.01} \right)^{1/4}$$

$r < 0.036 \Rightarrow V^{1/4} < 1.4 \times 10^{16} \text{ GeV}$. A detection of r would measure the energy at which inflation happened.

Tachyon inflation: slow roll in a few lines

Standard FRW, $\rho + p = \rho \dot{\theta}^2$ and $\dot{H} = -4\pi G(\rho + p)$:

$$\varepsilon_1 = -\frac{\dot{H}}{H^2} = \frac{3}{2} \dot{\theta}^2 \quad (\text{exact}).$$

Slow roll: $3H\dot{\theta} \simeq -V'/V$, $H^2 \simeq \frac{8\pi G}{3}V$, so

$$\varepsilon_1 \simeq \frac{M_{\text{Pl}}^2}{2} \frac{V'^2}{V^3}, \quad N \simeq \frac{1}{M_{\text{Pl}}^2} \int_{\theta}^{\theta_{\text{end}}} \frac{V^2}{|V'|} d\theta$$

Compare the canonical field: $\epsilon_V = \frac{M_{\text{Pl}}^2}{2} V'^2/V^2$. The extra $1/V$ means the same potential gives different (n_s, r) .

Braneworld makes inflation longer

RSII with $V \propto \theta^{-4}$, dimensionless units, $\kappa^2 = 5$, $\theta_0 = 0.25$:

- standard tachyon: $N \simeq \kappa^2/8\theta_0^2 - 1 \approx 9$
- RSII: $N \simeq \frac{\kappa^2}{8\theta_0^2} \left(1 + \frac{\kappa^2}{36\theta_0^4}\right) \approx 330$

The ρ^2 term in the brane Friedmann equation adds Hubble friction.

Holography in one slide

- A *hologram* stores a 3D image on a 2D film.
- Holographic principle ('t Hooft, Susskind): the information in a region is bounded by its *area*.
Example: black-hole entropy $S = k_B A / 4 \ell_P^2$.
- AdS/CFT (Maldacena 1997): a gravity theory in 5D anti-de Sitter space is *equivalent* to a conformal field theory (no gravity) on its 4D boundary.

RSII vs. holographic braneworld

Same AdS_5 bulk. In RSII the brane sits inside the bulk. In the holographic model it sits *at the boundary*, and the bulk acts on it through the boundary CFT. Result: the h^4 term in the Friedmann equation, a maximal density, and no initial singularity.

Hubble tension: two ways to measure H_0

Late Universe: distance ladder

1. Parallax (Gaia) calibrates Cepheids in the Milky Way.
2. Cepheids calibrate SNe Ia in nearby galaxies.
3. SNe Ia in the Hubble flow give v vs. D .

Direct, almost model-independent: $H_0 \approx 73$.

Early Universe: a standard ruler

1. Sound waves travel $r_s \approx 147$ Mpc (comoving) before recombination.
2. The CMB measures its angle $\theta_* = r_s/D$ to 0.03%.
3. Λ CDM gives $r_s \Rightarrow D \Rightarrow H_0 \approx 67$.

Precise, but it assumes the model.

Where new physics could hide

Early dark energy adds energy just before recombination, shrinks r_s , and so raises the inferred H_0 . Or the ladder has an unknown systematic. JWST is now re-checking the Cepheids.